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THE STORY OF HALLEY'S COMET.

JOHN CANDEE DEAN.

FOR POPULAR ASTRONOMY.

After an absence of seventy-five years, and after visiting a region that is perhaps fifty million miles more distant than the outermost planet of the solar system, Halley's comet is again approaching us, and even now, is nearer to us than the planet Saturn.

The return of this celebrated comet—the first known to move in a closed orbit—causes it to be an object of extraordinary attention. Its brilliancy, its sensational size, the records of its returns, extending back nearly two thousand years—the consternation spread throughout the world by the belief that it would destroy the Earth, make it the most famous comet in history.

In 1682, during the reign of Charles II, a comet appeared of extraordinary size which was observed by Newton, Halley and other astronomers of the time. Halley followed its course among the stars and comparing his observations with the records of previous comets, came to the conclusion that the comets of 1456, 1531 and 1607 were but different appearances of the same object. He staked his reputation on a prediction that the comet would return in about seventy-five years. True to this prediction, it did appear in 1758, when Halley had been sleeping in his grave for sixteen years. The reason that the name of Palitsch, a Saxon peasant has been preserved to posterity is, that his eye was the first to catch sight of the returning comet.

Comets have always been considered precursors of war and other horrible woes. This belief, even now, is not confined to the unlearned. Once it was universal. In the year 43 B. C. a comet appeared that was so bright it could be seen in the daytime. This was the year following the assassination of Julius Caesar, and the Romans believed it to be his metamor-

phosed soul armed with fire and vengeance. No wonder that the great comet added to the confusion following the death of Caesar. Rome was without a ruler. Civil war added to the prevailing consternation. Octavius was playing with the senate, waiting for his opportunity to assume control. Caesar's assassins had either been killed or driven from Rome. Brutus and Cassius fell at Philippi and the delicate neck of Cicero had been severed by a single stroke of a Roman soldier's sword.

FIGURE 1
HALLEY'S COMET OF 1682.

When the great die, it is natural to think that all nature is convulsed.

"A little ere the mightiest Julius fell,
The graves stood tenantless, and the sheeted dead
Did squeak and jibber in the Roman streets:
As stars with trains of fire and dews of blood,
Disasters in the Sun; and the moist star;
Upon whose influence Neptune's empire stands,
Was sick almost to doomsday with eclipse."

Calphurnia had warned Caesar of his impending fate by saying—

"When beggars die there are no comets seen;
The heavens themselves blaze forth the death of princes."

In 1680, only two years before the appearance of Halley's

comet, there blazed forth a comet that was remarkable for being the one which Newton first proved to be moving under the Sun's attraction. It was also notable for having been investigated by Halley, who found that this brilliant celestial body had appeared at intervals of about five hundred and seventy-five years, of which this was the fourth recorded return. Its first appearance, which closely followed the death

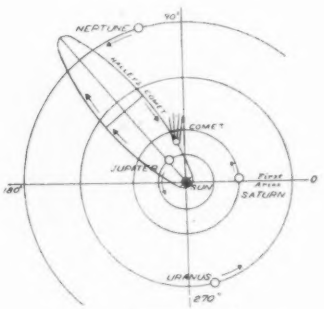

FIGURE 2
ORBIT OF HALLEY'S COMET.
Showing position of the major planets for July 1st, 1908 and probable position of Halley's Comet on that date.

of Caesar, has just been referred to; its second visit was in the year 531; its third in 1106; its fourth in Halley's time; and its fifth may be looked for by our expectant posterity about the year 2255.

When Halley traced the orbit of his comet of 1682, he discovered it coincided with that of the comet observed by Kepler in 1607. So close were the orbits together, that if drawn in the heavens, the human eye could almost see them joined in a single line. He found that it was also true of the comet seen in 1531. This was conclusive proof that these bodies were identical. It is said that from the testimony of early writers, Halley traced the returns of his comet back for seventeen centuries.

One of the early appearances of Halley's comet was in 1066, and because this was the year of the Norman Conquest of England, unusual attention was drawn to it. To William, the Conqueror, it was an auspicious sign in the heavens, but to Harold and his army it proved a portentous omen.

In the library of the city of Bayeux, France, is preserved a remarkable piece of tapestry of great archaeological interest

by reason of the details shown of arms and costumes. It is embroidered with episodes of the Norman Conquest from Harold's visit to the Norman court, to his death at the Battle of Hastings, and is said to have been made by Matilda, queen of William the Conqueror. On it is figured Halley's comet of 1066.

On the return of this comet in 1456, a wider terror was spread than ever known before. It was described by those who saw it as an object of unheard of dimensions, with a tail that stretched more than a third of the way across the heavens. The belief was general among all classes that the comet would destroy the Earth and that the Judgment Day was at hand. Three years before, the victorious Turks had captured

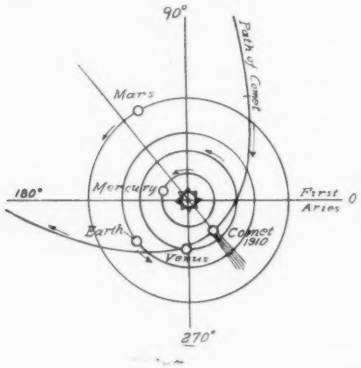

FIGURE 2.
ORBIT OF HALLEY'S COMET.
Showing the position of the Terrestrial planets on May 1st, 1910 and probable perihelion position of Halley's Comet on that date.

Constantinople. This year Mohammed II, surnamed "The Conqueror," crossed the Hellespont with his army and threatened to overrun all Europe. The terror was such that the people seemed regardless of the present and anxious only for the future. They gave up all hope and prepared for their doom. Treasure uncounted was poured into the apostolic chamber, by the frightened people. Alfonso Borgia was pope, under the title of Calixtus III. He ordered the Ave Maria repeated three times a day instead of two, and to the prayer was added—"Lord save us from the Devil, the Turks, and the Comet." He ordered the church bells rung at noon, which was the beginning of a practice still common in Christian countries.

FIGURE 4.
DONATI'S COMET OF 1858.

At length the fiery comet began to wane; a victory had been achieved over the demon in the firmament, the Turks were checked by Hunyady at Belgrad, but tranquility was not fully restored until the final disappearance of the comet.

Halley's comet of 1682 was larger and brighter than that of 1456. Its tail was more than ninety-six million miles in length, but science had robbed it of its terrors, and history pointed to the failure of its predecessor. Newton had just discovered the law of universal gravitation, which explained the motion of planets and comets around the Sun and placed the solar system on a mechanical basis.

FIGURE 5.
HEAD OF DONATI'S COMET.

The tendency of a superstition to persist, even after closely allied phenomena have been explained on a purely natural basis, is illustrated in the belief that planetary motion was due to guiding spirits. Although Copernicus discovered that the planets revolved around the Sun, instead of the Earth, he still believed that their motion was controlled by these spirits. Galileo confirmed the truth of the heliocentric theory, but his faith in the supernatural motion of the planets was undisturbed.

Not until the genius of Newton had discovered the law of universal gravitation, and provided a mathematical foundation for Kepler's laws, were these conducting spirits entirely dis-

missed. It required the discoveries of three great men and two centuries of time to disprove the foolish fancy of little men.

Edmund Halley was a great astronomer who had an illustrious career. He was secretary of the Royal Society, professor of geometry at Oxford, and Astronomer Royal.

William III placed him in command of a ship for scientific exploration, more especially for the purpose of studying the variation of the magnetic needle, and also ordered him to visit the American colonies to make such observations as were necessary for the more accurate determination of their longitude and latitude. Halley's constant gaiety, cosmopolitanism, and good fellowship, won for him great popularity with his colleagues, while his reputation as a scientist was earned by the most arduous and protracted labor.

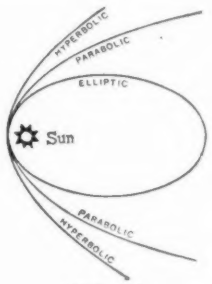

FIGURE 6.
ORBITAL CURVES OF COMETS.
Periodic comets move in elliptical orbits. Comets of higher speed move in parabolic orbits. Comets of the highest speed move in hyperbolic orbits. The last two curves carry them off into the wilderness of space, never to return.

His career was closely interwoven with that of Newton's. It is related that Halley, Hook and Sir Cristopher Wren were at London debating about the forces controlling planetary motion. Finally Halley announced that he would go down to Cambridge and present the question to the great Newton. Going direct to the philosopher's rooms at the University, he at once asked—"What orbit will a planet move in, if attracted by the Sun inversely as the square of the distance?" "In an an ellipse" said Newton. "How do you know that?" said the astonished Halley. "I have calculated it" said Newton. He then told of many more wonderful discoveries, and, to Halley's joy, informed him that they were all written out in a manuscript which he would give him to read; but to Halley's dis-

may the volume could not be found. It had been mislaid, and he returned to London without it.

Afterward Newton reproduced his manuscript and forwarded it to Halley. He found that it was an orderly and scientific unfolding of the achievements of the greatest scientist that the world had produced. It satisfactorily solved all the great problems of astronomy. It showed that comets like the planets, revolve in regular orbits. It gave to the world the great discoveries for which mankind had long been waiting, and furnished a foundation for modern thought and modern science.

Perhaps Halley's greatest achievement was in rescuing this work from oblivion, obtaining the consent of the author to

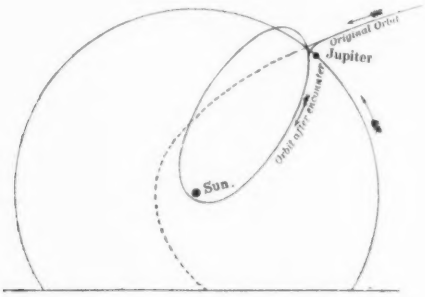

FIGURE 7.
SHOWING HOW JUPITER ENTRAPS COMETS.
The original orbit of comet is indicated by the dotted line. When the comet happens to pass near Jupiter, it is drawn out of its course and takes the new elliptical orbit shown, the outer end of which is near Jupiter's orbit.

publish, paying the expense himself of publication and personally seeing the work through the press. It is known as Newton's "Principia" and is said to be incomparably and indisputably the greatest scientific work ever published.

The discovery that Halley's comet was periodic established the fact that many comets are members of the solar system and governed by the same laws of motion as the Earth and other planets. Their orbits, however, are flat ellipses and their planes of revolution bear no relation to the ecliptic.

If a periodic comet were the only member of the solar system, its path would be an exact ellipse and its period of revolution unchangeable, but, owing to gravitational attraction of the great outer planets, comets are sometimes drawn out

of their courses, which affects their time of revolution around the Sun; hence, it becomes necessary to compute each return separately. The last return of Halley's comet was in 1835. Four great astronomers had figured its orbit, and assigned November as the time for its nearest approach to the Sun. This proved to be correct. It passed its perihelion on November 16th and was visible from August 5th, 1835 to May 5th, 1836. The late Compte de Pontecoulant published the result

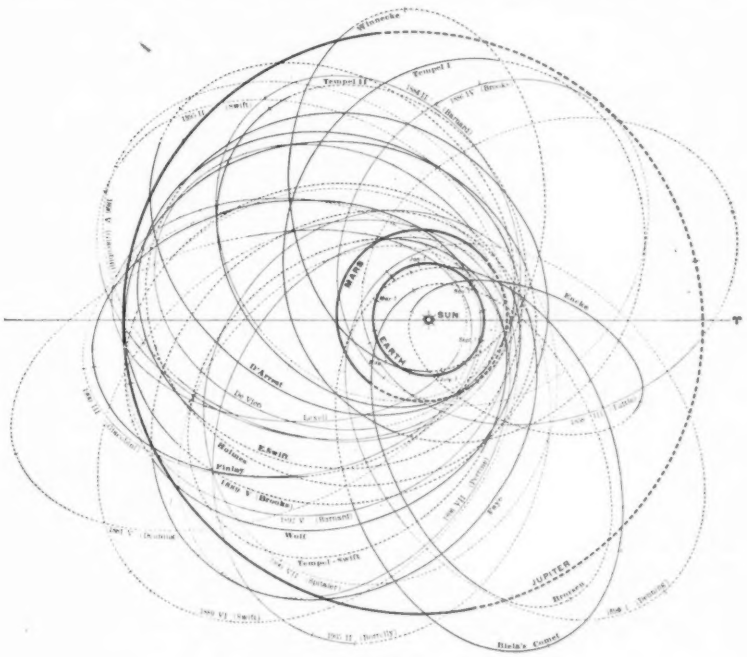

FIGURE 8.
JUPITER'S FAMILY OF COMETS.

of his computations in 1864 and gave May 1910 as the time for the comet's next perihelion passage. It is quite probable that photographs of Halley's comet will be obtained before the end of this year. Figure 10 shows the apparent positions of the comet in the heavens for every three months, from July 1st, 1908 to October 1st, 1909, after which time it will probably be visible to the naked eye.

From what has been said, it will be observed that the period of Halley's comet is about seventy-five and a half years. Figure

2 shows that its orbit extends beyond Neptune's; in fact, at aphelion it is five hundred million miles farther from the Sun than Neptune is, while at perihelion it is thirty-eight million miles nearer to the Sun than the Earth is. The Earth's orbit is too small to appear in this figure, and Figure 3, drawn on a much larger scale, is introduced to show the comet's orbit when near the great luminary.

For the past thirty-five years, Halley's comet has been traveling toward us, and it is now nearer to us than Saturn. The plane of the comet's orbit is at an angle of seventeen degrees to the plane of the Earth's orbit, and its motion is retrograde, or in a direction opposite that of the planets. All the planets move eastward, while Halley's comet moves westward. It must not be assumed that its orbit is so large that it extends into the region of the fixed stars. The distance to the nearest of the fixed stars is so vast that a comet moving under the Sun's attraction from that region would require twenty-million years for a complete revolution.

Not more than one comet in five is periodic. When the flight of a comet is more than twenty-six miles a second, at a distance of ninety-three million miles from the Sun, its orbit will be either a parabola or a hyperbola, and after passing around the Sun, these curves will carry it off into limitless space never to return. If, however, the speed is less than stated above, the orbit will be elliptical, and as the ellipse is a closed curve, the comet will re-appear at regular intervals.

Certain periodic comets have had their orbits changed from parabolas to ellipses by the disturbing attraction of the outer planets. This problem was worked out by Laplace, who found that when an irregular comet passed closely in front of a large planet, its velocity would be retarded to such an extent that it would fall into an elliptical orbit and become periodic. If the planet were Jupiter it would be said that the comet had been "captured" by Jupiter, and belonged to "Jupiter's family." Its aphelion point would then remain near Jupiter's orbit. Neptune has a family of six comets including Halley's; Uranus a family of three; Saturn two, and Jupiter about thirty. Figure 8 shows the orbits of Jupiter's family of comets. It is large because its mass is greater than that of all the other planets combined, and its influence over comets is proportioned to its mass. The individuals of Jupiter's family are small and their periods of revolution run from three to nine years.

Large comets are magnificent objects, as bright as Venus, with nebulous heads sometimes as large as the Moon, and luminous trains extending one-third of the way across the sky. The most celebrated comet of the nineteenth century was Donati's, which attained its greatest brilliancy in October 1858. Its orbit is elliptical, and its period of revolution around the Sun is nearly two thousand years. This comet is shown in Figure 4, and its remarkable head, from the drawing made by Bond (who was director of Harvard Observatory at the time of its appearance) is shown in Figure 5.

Comets consist of three parts—

1st—The nucleus, or bright starlike kernel, which is the real comet, and from which the other parts are evolved.

2nd—The coma or hazy envelope surrounding the nucleus.

3rd—The tail or luminous train stretching away into space. Sometimes the tail is entirely wanting or hardly discernible.

It is a strange fact that the tail follows the comet as it approaches the Sun and goes before it when it recedes from the Sun. The tail develops as the comet draws near the great luminary, and diminishes in size and splendor as it moves away from it. On the contrary, the nucleus and coma always contract when near the Sun and expand again as the comet recedes.

The mass or weight of comets is small compared with other heavenly bodies, but their volume is much larger than any member of the solar system. The mass assumed by Young for the very largest comet, like Halley's, is equal to a ball of iron 150 miles in diameter. The tail is so unsubstantial as to be below the density of the best vacuum scientifically obtained. It is millions of times less dense than our atmosphere.

Comets have neither tails nor light until they approach the Sun. The tails are evolved by the stimulus of the Sun's heat. They become luminous by solar light and heat. When a distant comet is discovered through the telescope, it is merely a round nebula, but as it approaches the Sun the nucleus appears and a luminous jet or tail developes.

The very recent discoveries in radio-activity and of light pressure, give a complete explanation of the nature of tails of comets. Professor Arrhenius, of the University of Stockholm, is the father of nearly all the theoretical interpretations of observed phenomena of mechanical pressure of sunlight.

The tails of comets at first show the presence of hydrocarbons, and as they approach still nearer to the Sun, metallic

vapors appear. The matter composing the tail is driven out of the nucleus by solar heat and is repelled by light pressure into space, where it passes away never to be recovered by the comet. The tail increases in size as the comet passes nearer to the Sun. Iron, magnesium, and other metallic vapors appear, and as the heat increases, the hydrocarbons break up into smoke or soot. The pressure of sunlight is probably due to the bombardment of corpuscles constantly flung from the Sun. This pressure is delicate for a given area, and for the whole Earth's surface amounts to only 70,000 tons; but very finely divided matter, like the soot of the comet's tail, is acted on energetically and driven out with enormous velocity. The Sun's powerful light illuminates the particles of free carbon and causes the tail to glow with a silvery light.

Closely associated with the subject of comets is that of meteors, or shooting stars, which occasionally penetrate the atmosphere and fall to the Earth. They are supposed to be fragments of comets, as will be shown presently. It is estimated that about five hundred meteoric stones annually strike the Earth's surface. Most of these are small, not weighing more than a pound or two, while some weigh hundreds of pounds. The largest known meteorite is that which was brought from the west coast of Greenland in 1897 by Lieutenant Peary, U.S.N. It weighs ninety-tons and is composed of 92 per cent iron, and eight per cent nickel. The only pure iron found on the Earth's surface is of meteoric origin. Meteors are black, cold and invisible until they enter our atmosphere. The friction due to shooting through the air at an incredible velocity of perhaps forty miles a second, fuses them to a white heat, causing them to glow with the brilliancy of the electric arc. Before they enter the atmosphere, they are called meteoroids; while luminous they are called meteors; after they strike the Earth they are called meteorites.

The mean velocity with which meteors enter the upper atmosphere is about twenty-six miles a second, or one-hundred times the muzzle speed of a rifle bullet. It is calculated that five hundred million meteors collide with the air every day and although their average size may not be larger than a buckshot, the enormous penetrating power of even the smallest meteoric grains would be sufficient to destroy all life were it not for the thick, soft, elastic, kindly envelope of air that surrounds us.

Nearly all meteors burn out and volatilize by air friction within a second of time, while still fifty miles from the Earth's

surface. Until the discovery of the mechanical theory of heat, the phenomenon of burning meteors was a stumbling block to investigators. Lord Kelvin showed that a thermometer placed in front of a rapidly moving body rose one degree when passing through the air at 125 feet per second and the temperature increased as the square of the velocity. A few grains of meteoric iron striking the atmosphere with the velocity of celestial

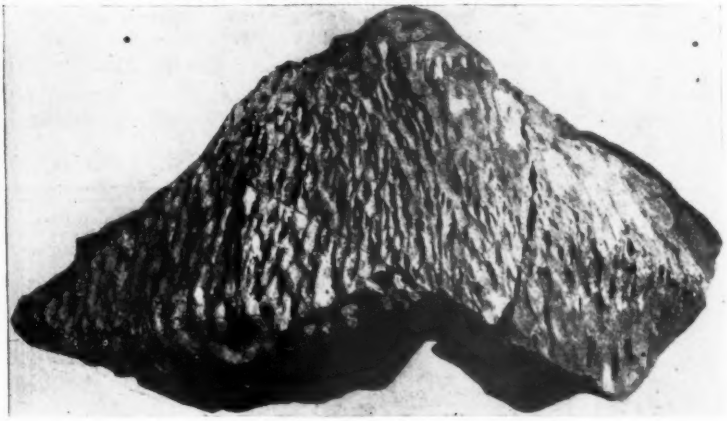

FIGURE 9.
A METEORITE. WEIGHT 700 lbs. PHOTO BY FARRINGTON.

matter evolves as much light and heat as the explosion of several pounds of gunpowder.

The very few large, firm meteors that pass through the atmosphere without being destroyed by potential heat, have their motion reduced to the moderate velocity of 400 to 500 feet per second. At Upsala, Sweden, fragments of a meteor struck the ice of a lake in winter and rebounded without breaking the ice.

It is believed that vast swarms of meteoroids revolve around the Sun. Under certain favorable conditions these meteoric rivers become visible to the naked eye. A few minutes after sunset, especially in the spring, an afterglow of white light, resembling the Milky Way, may be seen extending from the western horizon along the Zodiac towards the zenith. It is conical in shape with the middle of its base where the Sun sets. It is called the zodiacal light and has been observed for ages. It is also visible in the early morning. In the Orient,

where there are religious ceremonies at the break of day, the light is called the "fox's tail" or "false dawn." It is supposed to be the reflected light from myriads of meteoroids revolving around the Sun, extending their course far beyond the orbit of Neptune, the outermost planer of the solar system.

When comets pass quite close to the Sun they are subjected to disintegrating influences, and have been known to divide even while under the observation of astronomers. Whenever a comet begins to break up, the disintegration continues until it becomes a mere group of cometary fragments moving in a great elliptical orbit around the Sun. Roche, Maxwell and others have discovered a law which explains the breaking up of comets. It is called the "Law of Disruptive Approach" and is substantially as follows:—

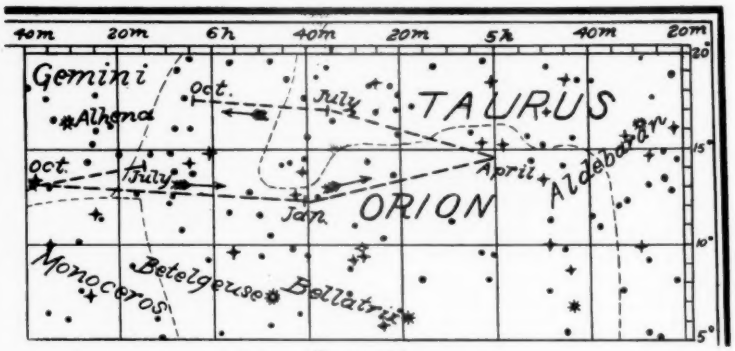

FIGURE 10.

COURSE OF HALLEY'S COMET.

The dotted lines connect the predicted positions of Halley's Comet for every three months from July 1st, 1908 to October 1st, 1909.

Whenever a celestial body passes within a certain distance of a larger, denser body, it will be torn into fragments by differential attraction. The critical distance varies with the relative sizes of the two bodies, and is called "The Roche Limit." This law not only explains the cause of the disruption of comets, but also accounts for the phenomenon of the rings of Saturn. By the disruptive approach of one or more of Saturn's Moons, within the Roche limit, they have been pulverized into meteors, which form the beautiful rings that revolve around the planet.

Mention has been made of gases being driven out from the nuclei of comets by the heat of the Sun, thereby con-

tributing material for the phenomenon of the tail. It has been found that meteorites are saturated with occluded gases, hydrogen, helium and carbon oxides. At one time Dr. Odling lighted up the theater of the Royal Institution with gas brought down from interstellar space by meteorites.

Because comets, unlike planets, observe no one direction in their orbits, but approach and recede from their great center of attraction in every possible direction, the danger of their collision with the Earth has raised a question of serious interest.

There is not the slightest doubt that if the head of a large erratic comet should strike the Earth, it would instantly reduce it to chaos, mingling its elements in ruin, which would result in universal death to nearly all forms of life. Should the nucleus of such a comet even sweep through the regions of our atmosphere, the resulting intensity of light would probably blind every eye that beheld it, and the gases evolved would vitiate the air so as to render it unsuitable for breathing.

The question of the danger of collision with comets was of such importance that the French government at one time called the attention of its ablest astronomers to the solution of the problem, and after mature examination they reported as follows:

"We have found that out of 281,000,000 of chances, there is only one unfavorable. Admitting then, for a moment, that the comets which may strike the Earth with their nuclei would annihilate the whole human race, the danger of death to each individual, resulting from the appearance of the unknown comet, would be exactly equal to the risk run, if in an urn there was only one single white ball among a total of 281,000,000 balls, and that his condemnation to death would be the inevitable consequence of the white ball being produced at the first drawing.

From an examination of Figure 3 some may infer that there is danger of Halley's comet clashing with some one of the planets. It must be remembered that this comet moves at an angle to the plane of planetary motion and loops through that plane at points where there is no possibility of collision.

Indianapolis, Indiana,
March 26, 1908.

THE CONSTANTS FOR THE EQUATOR.

ROBERDEAU BUCHANAN.

FOR POPULAR ASTRONOMY.

These constants as they are called are employed for the purpose of computing the rectangular coördinates of a planet in order to find the Right Ascension and Declination directly from the elements; but if applied to the major planets, they are very liable to lead the computer into one or more mistakes which would vitiate his results, and which it is the object of this article to point out. And moreover the astronomies which give these formulæ do not allude to this fact in any way.

The source of error is first, that this method does not take account of the perturbations of the heliocentric latitude, and the second is, what quantity is to be considered as the perturbations? This is also very likely to lead to another error.

It will perhaps make this matter more clear to give all the formulæ. The usual formulæ which are rigorously correct in all cases are as follows:

First Group: Rigorous formulæ

$$\lambda = \iota + \text{Equation of the Equinoxes} \qquad (1)$$

$$\left.\begin{aligned} x &= r \cos\beta \cos\lambda \\ y &= r \cos\beta \sin\lambda \cos\epsilon - r \sin\beta \sin\epsilon \\ z &= r \cos\beta \sin\lambda \sin\epsilon + r \sin\beta \cos\epsilon \end{aligned}\right\} \qquad (2)$$

The formulæ for the Constants for the Equator are as follows, their deviation may be found in Watson's Theoretical Astronomy, pp. 88-93, under Admiral Davis' Appendix of Gauss' Theoria Motus.

Second Group:

$$\Omega = \theta + \text{Equa. Equinoxes} \qquad (3)$$

$$i = i_1 + \Delta i \qquad (4)$$

$$\tan E = \frac{\tan i}{\cos \Omega} \qquad (5)$$

$$\left.\begin{aligned} \cot A &= -\tan\Omega \cos i & \sin a &= \frac{\cos\Omega}{\sin A} \\ \cot B &= \frac{\cos i \cos(E+\epsilon)}{\tan\Omega \cos E \cos\epsilon} & \sin b &= \frac{\sin\Omega \cos\epsilon}{\sin B} \\ \cot C &= \frac{\cos i \sin(E+\epsilon)}{\tan\Omega \cos E \sin\epsilon} & \sin c &= \frac{\sin\Omega \sin\epsilon}{\sin C} \end{aligned}\right\} \qquad (6)$$

And the coördinates are

$$\left.\begin{aligned} x &= r \sin a \sin (A + u) \\ y &= r \sin b \sin (B + u) \\ z &= r \sin c \sin (C + u) \end{aligned}\right\} \qquad (7)$$

As the equation of the equinoxes is added to the longitude ι in equation (1) to reduce the latter as given in the *Nautical Almanac* from the mean equinox of January 1 to the equinox of the given date, so it must be added to the node θ Equation (3) in the second group.

The latitude β_0 given in the tables of the planets as shown in a previous article in POPULAR ASTRONOMY XV, p. 140 is computed as follows:

$$\sin \beta_0 = \sin i \sin u \qquad (8)$$

which is generally tabulated. Then in using the Tables:

$$\beta = (\beta_0 - C^1) + pp + (\text{Pert} + C^1) + \text{sec var} \qquad (9)$$

We will now compare these equations last given which belong to the first group with the equations of the second group. The constant C^1 (ibid. p. 141,) is merely for the convenience of using the tables; it is the sum of all the constants added to each table of perturbations, in order to make all the quantities positive; and to keep up the equality it is deducted from the principal term of the longitude radius vector or latitude as the case may be. It is sometimes omitted. The proportional part *pp* is merely the interpolation from the tabular values to the exact value of the argument. The secular variation is taken account of in the second group by the change Δi in the inclination of the orbit i which is given in the Tables as the change in 100 years to be multiplied by the fraction of a century after 1900. The remaining term, the perturbations, it is seen are not taken account of in the second group. It will be noticed that in equation (2) that they do not affect the coördinates x; since a small change in β would have very little effect upon its cosine but this affects y and z through the term sin β. We can easily ascertain what effect the perturbations have upon the other coördinates by differentiating the two latter of equations (2) with β as the variable in the second member.

$$\left.\begin{aligned} dy &= -r \sin \beta \sin \lambda \cos \epsilon d\beta - r \cos \beta \sin \epsilon d\beta \\ dz &= -r \sin \beta \sin \lambda \sin \epsilon d\beta + r \cos \beta \cos \epsilon d\beta \end{aligned}\right\} \qquad (10)$$

The results being generally very small the first terms in the second member containing sin β are generally insignificant and may be neglected; cos β may be placed equal to unity, and we have left:

$$\begin{aligned} dy &= -r \sin \epsilon d\beta \\ dz &= \quad r \cos \epsilon d\beta \end{aligned} \qquad (11)$$

in which $d\beta$ is to be taken as the perturbations of the latitude and dy, dz, are its effects upon the coördinates. But for practical use, these formula must be still further transformed. Since $d\beta$ is given in arc, while y and z are given in parts of radius the second members must be multiplied by sin 1″ and as ϵ changes very slowly we may for a number of years to come, take it as 23° 27′ with its sine and cosine constant and multiplying the second number by 10^7 to give the results in units of the seventh decimal of logarithms we have, combining the constants and giving the result by its logarithm

$$\begin{aligned} dy &= -[1.285]\, r d\beta \\ dz &= \quad [1.648]\, r d\beta \end{aligned} \qquad (12)$$

And here the second source of error may arise, $d\beta$ must be taken as purely the perturbations, freed from secular variation of the latitude which is generally combined with them in using the tables, and if a constant has been added to the tables of the latitude, it must be deducted; dy and dz are then the effects of the perturbations upon the coördinates y and z to which they are to be algebraically added.

In case the perturbations of the planet may be large, equation (10) should be examined to ascertain whether the omitted terms become appreciable.

These equations look formidable, but they are quite easy, since the quantities vary slowly and may be computed for an interval of five or ten days, using seven place logarithms.

The constants for the equator were first used for finding the geocentric places of newly discovered asteroids. In this case there are no perturbations to be considered; and the equations of the equinoxes may be omitted at first, to be taken account of at the end of the work. The constants then vary quite regularly and may be computed for an interval of at least one year if necessary, and interpolated for the intermediate dates. The elements of the asteroid are supposed to have been computed, and the work then proceeds as follows.

The mean anomaly M is found from the mean motion and the epoch date; the eccentric anomaly E from Kepler's equa

tion $M = E_1 - e \sin E$; and the true anomaly v and distance r, from the equations

$$\sqrt{r} \sin \tfrac{1}{2} v = \sqrt{a(1+e)} \sin \tfrac{1}{2} E$$
$$\sqrt{r} \cos \tfrac{1}{2} v = \sqrt{a(1-e)} \cos \tfrac{1}{2} E$$

Then u is given by the following

$$u = v + \pi - \Omega.$$

With the constants for the equator the coördinates x, y, z, may now be computed and the geocentric place derived by the usual formulæ. Finally the results may be reduced to the equinox of the date by applying the equation of the equinoxes in right ascension and declination which may be taken from the *Nautical Almanac*. Professor Watson in his Theoretical Astronomy gives an example of this work, with also a method of solving Kepler's equation.

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THE ART OF NUMERICAL CALCULATION.

F. H. SEARES.

FOR POPULAR ASTRONOMY.

The circumstances affecting the numerical solution of problems are so different from those accompanying analytical investigations that the qualifications of the computer are quite distinct from those of the mathematician. Yet they include something more than the ability to combine numbers with rapidity and precision, for the practice of the art of computation requires, in a high degree, judgment, while the dexterous manipulation of figures depends mainly upon a specialized development of the memory. The possession of one of these faculties by no means implies the existence of the other. Indeed the extraordinary memory for figures exhibited by such prodigies as Dase, Mondeux, and Inaudi is frequently accompanied by a deficiency in those qualities necessary for the practice of the art in its broader sense.

It is the purpose of this paper to indicate the origin and nature of the requirements of the computer, and to formulate certain details important for the numerical solution of problems occurring in scientific investigations.

I

In order to understand the nature of the qualifications of the computer, we must consider, first, the origin and character of the data entering into computations; second, the distinctive features of the aids used by the calculator for the facilitation of his work; and, finally, the limitations of mathematical expressions when used as a basis for quantitative investigations.

The data used by the computer are usually the result of observation and experiment, and as such, are affected by errors of observation. Errors of observation, carefully to be distinguished from mere blunders, such as the incorrect reading of a circle by a whole degree or an exact number of minutes, arise from a variety of causes. The limitations of the observer's senses, defects of construction and adjustment in the instrument, and variations in the temperature of the surrounding air cause the final result to differ from the value sought by the observer. Certain disturbing factors can be neutralized by a special arrangement of the observing program; others can be minimized by care and the use of instruments of high precision; but even when instrument maker and observer have done their best there remains in the final result an uncertainty beyond control, whose magnitude varies with the nature and circumstances of the observations. It may amount to a considerable fraction of a foot, as in the measurement of a long line with the surveyor's chain, or it may be only a few billionths of a millimeter, as in the precise determination of the wave length of a line of the solar spectrum. It may be a minute of arc, as in the measurement of the altitude of a star with the engineer's transit, or only one or two tenths of a second, as in the precise determination of latitude with the modern zenith telescope.

The aids employed by the computer for the facilitation of his work are numerous. That most commonly used is the ordinary logarithmic-trigonometric table, and the remarks which follow apply particularly to this contrivance, although they are true, in a measure, of the various other aids used to lessen the labor of numerical calculation.

The quantities whose numerical values are contained in a

table of logarithms can not be expressed exactly in the form of a decimal fraction. The difference between the true value and the numerical expression for the same can be reduced to any assigned limit by sufficiently increasing the number of decimals, but, however great this number, the representation will not be exact. The tabular values of the logarithms are therefore approximations. The approximations of a 7-place table are of a higher order than those of a 5-place table, but they are, nevertheless, approximations. Further, the computer usually requires, not the printed values of the function, but those corresponding to intermediate values of the argument. Thus, he may desire the logarithm of the sine of 26° 18′ 36.″4 to six places of decimals, a quantity not given in any logarithmic table. The ordinary 6-place table contains the logarithms of the trigonometric functions for every 10″, and the required logarithm must be found by interpolation from the tabular values for 26° 18′ 30″ and 26° 18′ 40″. This introduces an additional uncertainty for the interpolation process is usually inexact. The total error in the interpolated quantity has a maximum value of one unit of the last place of decimals, although its average is less. Generalized, this result may be expressed as follows:

$$\mathrm{E}\,[f(x)] = 10^{-r}\,. \qquad (1)$$

The symbol E followed by $f(x)$ in brackets is a general notation signifying the maximum error in $f(x)$. For the case considered, $f(x)$ represents a quantity interpolated from a table of r places with the argument x. For inverse interpolation, namely, that which derives the value of x corresponding to a given $f(x)$, the maximum error of the interpolated quantity is

$$\mathrm{E}\,[x] = 10^{-r}/2\,f'(x) \qquad (2)$$

where $f'(x)$ is the first derivative of the tabular function $f(x)$.

From (1) it will be observed that the maximum error affecting the result of a direct interpolation is independent of the nature of the function and the part of the table used, and varies only with the number of decimals employed. For inverse interpolation the error varies not only with the number of decimals, but also with the nature of the function and the magnitude of the quantity x. As an illustration, consider the following special cases: $f(x) = \log x$, $f(x) = \log \sin x$, $f(x) = \log \cos x$, $f(x) = \log \tan x$. The application of (2) gives,

when x is interpolated from log x, $\quad E[x] = 10^{-r} \frac{x}{2M}$,

" " log sin x, $\quad E[x] = 10^{-r} \frac{\rho''}{2M} \tan x$,

" " log cos x, $\quad E[x] = 10^{-r} \frac{\rho''}{2M} \cot x$,

" " log tan x, $\quad E[x] = 10^{-r} \frac{\rho''}{4M} \sin 2x$,

in which $M = 0.434\ldots$, the modulus of the common system of logarithms, and $\rho'' = 206265''$, the number of seconds in the radian. Thus, the maximum error in a number interpolated from its logarithm is proportional to the number itself; and the maximum errors in an angle, x, derived from log sin x, log cos x, and log tan x are respectively proportional to tan x, cot x, and sin $2x$. For the case of the sine and cosine they may be very large owing to the presence of the tangent and cotangent as factors. The last three relations express analytically the well-known fact that the value of an angle can be determined with greater precision from the logarithm of its tangent than from that of its sine or cosine.

Finally, the numerical solution of a problem is accomplished by substituting into a mathematical expression the values of the data corresponding to the problem in question. This involves the combination of a series of numbers which are approximations, for, as we have seen, both the data of observation and the quantities derived from the tables of logarithms are uncertain. The final result must therefore be an approximation. Its error arises partly from the errors in the data and partly from the limitations of the tables. That part arising from the former source we may call the Resultant Error of Observation, while that originating in the limitations of the tables is known as the Accumulated Error of Calculation. The magnitude of these errors depends largely upon the nature of the formula which expresses the analytical solution of the problem. This may be such that the errors of observation enter into the final result greatly reduced in magnitude. For example, in the determination of the value of one division of a scale by a comparison with a second scale of known length, we have for the required quantity an expression of the form

$$d = \frac{r_n - r_0}{n}$$

in which r_n and r_0 are readings from the second scale corresponding to the 0th and the nth divisions of the first. What-

ever the magnitude of the errors of observation, they enter into the final result by only $1/n$th of their amount. On the other hand, quite the reverse may be true. Thus, in the calculation of the ratio a/b, where $a>b$, and b itself is affected by an uncertainty, an error of observation if you will, the resultant error of observation will exceed the error affecting b approximately in the ratio of a to b^2. With the errors of calculation the case is different. Here there is almost always a certain multiplication of error, so that the accumulated error of calculation is usually in excess of the uncertainties attached to the individual numbers which enter into the computation; and, generally speaking, the longer the calculation, the greater will be the accumulation or multiplication.

Again, the solution of a given problem is frequently capable of expression in a variety of ways. Analytically considered, these may be identical, but viewed from the standpoint of practical applications, they may present the greatest diversity. For example, the two expressions

$$y = 2 \sin^2 1/2\, x \tag{3}$$

$$y = 1 - \cos x \tag{4}$$

are theoretically equivalent, but when used for the calculation of values of y corresponding to given values of x, they are by no means identical, especially for values of x near $0°$. As an illustration, consider the final errors resulting from an r-place calculation of (3) and (4) for the determination of y corresponding to $x = 2° \pm 2'$, where we may think of x as the result of an observation whose uncertainty is expressed by the appended quantity $\pm 2'$. An appropriate investigation shows that their maximum values are

$$\mathrm{E}\,[y] = 0.00002 + 0.003 \times 10^{-r}, \tag{3a}$$

$$\mathrm{E}\,[y] = 0.00002 + 3 \times 10^{-r}. \tag{4a}$$

The first terms in the right members are the resultant errors of observation. The last are the accumulated errors of calculation. So far as the precision to be obtained with a specified number of decimals is concerned, the advantage is obviously in favor of equation (3).

With these facts before us we are in a position to appreciate better the qualifications required of the computer. His aim must be so to arrange the calculation that the errors in the data and the errors of calculation will produce the

minimum possible effect upon the final result, and, at the same time, to derive this result with the least possible expenditure of time and labor. It is evident that his task is one of some complexity. The conditions to be satisfied are, to a certain extent, contradictory. For example, the accumulated error of calculation can be reduced to any desired limit by sufficiently increasing the number of decimal places employed; but any such increase carries with it a notable increase in the labor of calculation. On the other hand, a reduction of labor can often be brought about by a modification of the formula to be calculated, but this in turn may involve a sacrifice of precision.

The adjustment of these variable factors to each other and to the requirements just expressed demands a nice balancing of detail, which becomes only the more difficult when it is considered that the problems presenting themselves for solution, and the conditions under which they arise, are the most diverse imaginable. It is obvious that the computer has to deal with questions whose answers are not to be discovered through the exercise of whatever skill he may possess in the manipulation of figures, however important this accomplishment may be for the technical performance of his labors. They can be found only in a detailed knowledge of what has been but outlined in the preceding paragraphs. In addition, the computer must ever be upon the alert with a discriminating judgment, if his work is to be consistent in its details, and economical of the time and energy required for its execution.

II

Leaving now the consideration of the subject in its general aspects, we proceed to a discussion of matters of more immediate practical significance.

(a) *General Arrangement and Procedure.*

Computations are most conveniently made upon cross section paper whose squares measure one-fifth or one-sixth of an inch on the side. Before any figures are entered, the symbols for the quantities to be combined should be written in a vertical column at the left of the sheet, care being taken to bring together, as nearly as may be, those symbols or arguments whose numerical values are to be combined. Even though the same quantity enter into the calculation at several points, write its argument but once. A very little practice will make

it possible to add or subtract numbers which are separated by several intervening quantities. The numbers are to be written in a vertical column immediately to the right of the column of arguments. If the same calculation is to be performed for a number of similar sets of data, the work should appear in parallel vertical columns, that for each set occupying a column by itself. In such a case do not complete the first column before beginning the others, but work across the page, inserting all the numbers corresponding to any given argument before proceeding to the others. If, however, several trigonometric functions of the same angle are required, all should be interpolated with a single opening of the table, even though their symbols occupy widely separated positions in the column of arguments. Further, in computing for similar sets of data, do not enter arguments for quantities which are constant for all the sets, but write the values of such constants on the lower edge of a card or slip of paper. This can be held above the numbers with which the constants are to be united and moved along from column to column as the additions or subtractions are performed. The beginner will proceed with the greatest security by writing the arguments fully and completely, although the experienced computer is able to abbreviate the work by omitting some of the arguments and performing the corresponding operations mentally. Thus, it is possible to form the sum of two logarithms, enter the table, and interpolate the corresponding number without writing down the result of the addition. The argument for the sum can therefore be omitted, but such abbreviations are to be introduced gradually, and only after some skill has been acquired.

Whenever it becomes necessary to abbreviate a number, say to r places, by dropping the higher decimals, increase the digit of the rth place by one unit when the neglected quantity exceeds one-half a unit of this place. If the decimals neglected are less than half a unit of the rth place, they are to be dropped without change in that place. When the neglected part is exactly a half unit of the rth place, it is a good rule to increase the digit of the rth place by one unit, in case that digit is odd; otherwise, drop the higher places without change in the rth place. Errors arising from the abbreviation will thus tend to neutralize each other in the long run.

(b) *Aids to the Computer.*

Machines for addition, multiplication etc. Their operation

is so simple that they require no special treatment in this place. Their construction is such that there is no accumulated error of calculation, unless the quantities involved are abbreviated by dropping higher decimal places.

The sliderule. In effect, this instrument is a graphical table of logarithms. In its usual form, the accumulated error of calculation generally amounts to a few units of the fourth place of decimals. It is, therefore, in nowise a substitute for tables of logarithms of 5, 6, and 7, or even 4 places. It is extremely convenient for certain classes of computation, but many experienced computers maintain that properly constructed tables of logarithms give more satisfactory results. In any case, its continued use results in a strain upon the eyes far greater than that accompanying the use of well printed tables.

Multiplication tables. The best are the *Rechentafeln* of Crelle, published by Reimer of Berlin. These tables give directly the exact products of numbers of three figures or less; and can be used for the determination of products of numbers of any magnitude. They can also be used for division, most conveniently, when the divisor is of three figures or less. Their only objection is their bulk. They should be in the hands of every computer.

Logarithmic-trigonometric tables. Those most generally useful are of five places of decimals, although 3, 4, 6, and 7-place tables are also frequently required, and should be within reach of all who have to deal with astronomical or geodetic calculations. In purchasing tables, care should be exercised, for many are badly arranged and unfit for the purpose for which they are intended. With the exception of 3-place tables, those not giving the differences of the adjacent logarithms, at least for the tables of trigonometric functions, should be avoided. The same is true of those not containing auxiliary tables of proportional parts. The tabulation of the logarithms of the trigonometric functions to six places of decimals for every minute of arc, only, is likewise a bad arrangement. It is also important that the tables for the sine and cosine should not be separated from those of the tangent and cotangent. And, finally, it is desirable to select tables containing addition-subtraction logarithms. There are numerous other points of minor importance, but a more detailed discussion can be replaced by the following list of satisfactory tables. The list does not pretend to be complete.

Four-place tables:
Slichter, Macmillan;
Bremiker, Weidmannsche Buchhandlung, Berlin.
Five-place tables:
Becker, Tauchnitz, Leipzig;
Gauss, Strien, Halle;
Albrecht, Stankiewicz, Berlin;
Newcomb, H. Holt & Co., New York;
Hussey, Allyn & Bacon, Boston;
Bremiker, Weidmannsche Buchhandlung, Berlin.
Six-place tables:
Bremiker, edited by Albrecht, Nicolaische Verlags-Buchhandlung, Berlin.
Seven-place tables:
Vega, edited by Bremiker, Weidmannsche Buchhandlung. This is the best 7-place table.
Bruhns, Tauchnitz, Leipzig.

In Bremiker's 4 and 5-place tables, the arguments for the trigonometric functions are expressed in decimals of a degree, the intervals being $0°.1$ and $0°.01$, respectively, for the body of the tables.

It is assumed that the student is familiar with the fundamental principles underlying the construction and use of the ordinary logarithmic-trigonometric tables. The details of their usage can therefore be dismissed with the following precepts:

(1) Do not use negative characteristics. When such occur, increase them by ten, and operate as though a minus ten were written after the logarithm. When two such logarithms are added, the sum will have an appended minus twenty, which should be reduced to minus ten, dropping at the same time ten units from the characteristic.

(2) In case the number corresponding to a given logarithm is negative indicate that fact by writing a subscript n after the logarithm. In combining a number of logarithms, affix a subscript n to the result when the number of n-logarithms is odd. If this is even, the resulting logarithm needs no subscript.

(3) Derive the logarithm of the secant and cosecant from those of the cosine and sine, respectively, by subtracting the latter from zero. This is most easily accomplished by subtracting each digit of the logarithm from 9, proceeding from left to right, until the last is reached, which is to be subtracted from 10.

(4) Interpolate all the functions required for any given angle

with a single opening of the table.

(5) In the formation of powers of numbers, care must be exercised when the power is fractional, and the number less than unity. After the logarithm of the number has been multiplied by the power, p, the appended characteristic, which is normally -10, will be $-10p$. This must be reduced to -10 by adding $10(1-p)$ to the characteristic proper and subtracting the same quantity from the characteristic appended to the result of the multiplication.

Addition-subtraction logarithms. The purpose of these tables is to determine the logarithm of $a+b$ when the logarithms of a and b are given. The following illustrates the principle underlying their construction and use:

Let

$$A = \log N,$$

$$B = \log (N+1),$$

where N represents any number. The addition-subtraction logarithmic table contains the values of B tabulated with the argument A.

Now suppose

$$N = \frac{b}{a}$$

whence

$$A = \log b - \log a,$$

and

$$B = \log \left(\frac{b}{a} + 1\right) = \log (a+b) - \log a,$$

or

$$\log (a+b) = \log a + B.$$

This is the fundamental equation for addition. The procedure is as follows:

Form $A = \log b - \log a.$

Interpolate B with A as argument.

Then,

$$\log (a+b) = \log a + B.$$

Again, let

$$N = \frac{a}{b} - 1,$$

whence

$$A = \log (a-b) - \log b,$$

and

$$B = \log \frac{a}{b} = \log a - \log b.$$

These are the fundamental equations for subtraction. The procedure is as follows:

Form

$$B = \log a - \log b.$$

Interpolate A with B as argument.

Then,

$$\log (a - b) = \log b + A.$$

The arrangement of the tables assumed $a > b$ for the solution of both the addition and the subtraction problem. The application of these tables involves the performance of one subtraction, one addition, and one interpolation. The use of the ordinary logarithmic table for the derivation of the same result involves three interpolations and one addition or subtraction. Further, as an illustration, in the 5-place tables of Gauss, the ordinary table covers 18 pages while the addition-subtraction table covers but 12, of which only 4½ are necessary for the addition problem. Finally, it can be shown that the uncertainty of a result derived from the addition-subtraction table is less than that accompanying the use of the ordinary table. From every standpoint, therefore, whether that of the number of operations to be performed, the number of pages to be thumbed, or the accuracy of the final result, the advantage is in favor of the addition-subtraction table.

Tables of squares, cubes, etc. Special tables. Barlow's Tables, containing the squares, cubes, square roots, cube roots, and reciprocals of all integers up to 10,000, is one of the most convenient. The roots are given to seven places of decimals, and the reciprocals partly to nine and partly to ten places. Some of the 5-place logarithmic tables, such as those of Gauss and Albrecht, also contains tables of squares. Any table of squares or cubes can be used inversely for the derivation of square and cube roots.

In addition to the various aids mentioned there are innumerable special tables designed for the solution of special problems. Almost any problem which has to be solved repeatedly for different sets of data can be simplified through the use of specially constructed tables. In this connection there is abundant opportunity for the exercise of ingenuity and skill on the part of the computer.

(c) *Resultant Error of Observation. Accumulated Error of Calculation. Number of Decimal Places.*

The estimation of the effect on the final result due to un-

certainties in the data, in other words, the evaluation of the resultant error of observation, is most conveniently made by means of the relation obtained by differentiating the formulæ to be solved with respect to the final result and the quantities whose values are given. The substitution of the uncertainties in the data for the corresponding differentials in this expression leads to a knowledge of the numerical value of the differential of the final result, which may be taken as the resultant error of observation.

The determination of the maximum possible value of the accumulated error of calculation for any given set of formulæ is a more or less complicated process. Since, however, the accumulated error seldom, if ever, reaches its maximum, a knowledge of its average magnitude is of more practical importance. This varies with the character of the equations to be solved, and its exact evaluation presents some difficulty. But for formulæ containing no critical features, such as abnormally large multipliers or small divisors, or differences defined by two relatively large and nearly equal quantities, or angles to be interpolated from sines or cosines, the following, based upon the theory of probabilities, gives an approximate expression of the average uncertainty in the logarithm of a result:

$$U_L = 0.4 \times 10^{-r} \sqrt{n} \qquad (5)$$

where r is the number of decimal places employed, and n the number of quantities involved in the calculation. If the final result is a number, N, its approximate average uncertainty will be given by

$$U_N = 10^{-r} N \sqrt{n}, \qquad (6)$$

or, if an angle, by

$$U_A = 0.4 \times 10^{-r} 206265'' \sqrt{n}. \qquad (7)$$

The following table shows the results given by (5), (6), and (7) for tables of 3 to 7 places, n being equal to unity. To obtain the accumulated error of calculation for any given case, it is only necessary to multiply the proper tabular value by the square root of the number of quantities entering into the calculation.

No. of Decimals $= r$	$n = 1$		
	U_L	U_N	U_A
3	0.4×10^{-3}	$10^{-3} N$	1.′4
4	0.4×10^{-4}	$10^{-4} N$	8″
5	0.4×10^{-5}	$10^{-5} N$	0.8
6	0.4×10^{-6}	$10^{-6} N$	0.08
7	0.4×10^{-7}	$10^{-7} N$	0.008

Experience shows that these results are in close agreement with the average values actually occurring in practice.

The determination of the number of decimal places to be used in any given calculation is a matter of great importance. If the number chosen is too small, the precision of the data will be sacrificed. If too large, much unnecessary labor will be expended—just how much, is suggested by the fact that the relative amounts of time required to execute a calculation with 4, 5, 6, and 7 places of decimals are approximately expressed by the numbers 1, 2, 3, and 5, respectively, i. e., five times as much labor is required to complete a given calculation with 7-place logarithms as would be required if only 4-place tables were used. In practice, the number actually to be employed is usually determined by the accuracy of the given data. If this is to be used to its full advantage, a sufficient number of decimals must be employed to make the accumulated error of calculation small as compared with the resultant error of observation. Having determined the amount of the latter, the above table affords such indications as are necessary for the choice. Thus, if the resultant error of observation for a certain calculation consisting of sixteen logarithms is of the order of 10″, we find that 5-place tables should be used. The multiplication of the values of U_A corresponding to 4, 5, and 6-place tables by the square root of 16 gives for the accumulated errors of calculation 32″, and 3″.2, and 0″.3, respectively. The first of these being in excess of 10″, shows that the use of 4-place tables would sacrifice the precision of the data. The last is unnecessarily small as compared with the resultant error of observation, showing that the use of 6-place tables would involve a needless amount of labor. The second value, on the other hand, indicates that 5-place tables will entail the minimum of labor consistent with the precision desired.

Again, required the number of decimals necessary for a calculation involving 25 logarithms, in which the resultant error of observation is 0.0001, the result itself being a num-

ber whose approximate value is 100. The expression for the accumulated error of calculation is

$$U_N = 10^{-r} \times 100 \times 5 = 5 \times 10^{2-r}.$$

To make this small as compared with the resultant error of observation, r must be taken equal to 7.

For formulæ free from the critical features mentioned in the second paragraph of this section, the resultant error of observation will usually be of the order of the uncertainties in the data. The choice of the number of decimals is then very simple. If the data consists of numbers, it is only necessary to choose a number of decimals greater by one than the number of significant figures in the given quantities. If, on the other hand, the data consists of angles, a glance at the tabular values of U_A affords the necessary information.

The above suggestions by no means cover all the cases which may arise in practice, but they give an indication as to the general method of procedure.

(d) *The Adaptation of Formulæ.*

The following general suggestions indicate the more important points to be borne in mind.

(1) Whenever possible, transform equations containing sums or differences of terms into expressions containing only products or quotients. Thus, the equations

$$\begin{aligned} \sin\delta &= \cos z \sin\phi - \sin z \cos\phi \cos A, \\ \cos\delta \cos t &= \cos z \cos\phi + \sin z \sin\phi \cos A, \\ \cos\delta \sin t &= \sin z \sin A, \end{aligned} \tag{8}$$

defining δ and t in terms of z, ϕ, and A can be reduced to the form

$$\begin{aligned} \sin\delta &= m \sin(\phi - M), \\ \cos\delta \cos t &= m \cos(\phi - M), \\ \cos\delta \sin t &= \sin z \sin A, \end{aligned} \tag{9}$$

by introducing the auxiliaries m and M defined by

$$\begin{aligned} m \sin M &= \sin z \cos A, \\ m \cos M &= \cos z. \end{aligned} \tag{10}$$

The solution of (10) and (9) thus replaces the solution of (8). Although the number of equations involved in (9) and (10)

is five as against three in (8), the calculation is usually simpler in arrangement and control.

(2) Whenever possible, calculated angles should be determined from the tangent. This does not mean that the formulæ are necessarily to be arranged so as to express explicitly the tangent of a required angle. As an illustration, M in (10) will be determined from its tangent, derived by subtracting $\log m \cos M$ from $\log m \sin M$, although the equations do not express the tangent of M explicitly. The same is true of the determination of t from the last two of (9). It is possible to replace equations (9) and (10) by a single group of three equations giving directly the tangents of M, t, and δ; but this is not to be recommended, for there is no saving in labor, and the use of (9) and (10) as they stand introduces symmetry into the arrangement of the work, and simplifies the determination of the quadrants and the control of the calculation.

(3) Avoid formulæ expressing a quantity as the difference of two relatively large and nearly equal numbers. The comparison of equations (3) and (4) made in a previous section illustrates the disadvantage connected with expressions of this type.

(4) When it is necessary to determine a quantity differing but little from a second quantity whose value is known, arrange the formulæ in such a way as to express the *difference* of the two. The calculated difference applied to the known quantity then gives the desired result. Developments in series are frequently useful in this connection. Thus, the geocentric latitude, ϕ', is given by the relation

$$\tan \phi' = (1 - e^2) \tan \phi, \qquad (11)$$

in which ϕ' is the astronomical latitude, and e, the eccentricity of a meridional section of the Earth. The numerical value of the latter is approximately 0.08, whence it follows that ϕ' differs but little from ϕ. In accordance with the above mentioned principle, (11) is replaced for the purpose of numerical calculation by the following equivalent expression

$$\phi' - \phi = -690''.65 \sin 2\phi + 1''.16 \sin 4\phi \ldots \qquad (12)$$

in which the neglected terms are insensible. The first term in the right member of (12) calculated with 5-place and the second, with 3-place logarithms, gives the same precision as 7-place tables used in connection with (11).

(5) Calculations can frequently be much simplified by the use of approximate formulæ. It is obviously permissible to

neglect those terms in an equation whose numerical values are small as compared with the resultant error of observation. One of the most common methods of introducing such simplifications consists in the substitution of the first terms of the developments in series of the sine, cosine, and the tangent of small angles for the trigonometric functions themselves. Since

$$\begin{aligned} \sin x &= x - \frac{x^3}{6} + \ldots \\ \cos x &= 1 - \frac{x^2}{2} + \ldots \\ \tan x &= x + \frac{x^3}{3} + \ldots \end{aligned} \tag{13}$$

it follows that the errors resulting from the substitution mentioned will be of the order of $x^5/_6$, $x^2/_2$, and $x^3/_3$, respectively. The evaluation of the errors in any given case is readily accomplished by means of the approximate relations

$$\begin{aligned} x^2 &= (x \text{ in degrees})^2 \times 1', \\ x^3 &= (x \text{ in degrees})^3 \times 1''. \end{aligned} \tag{14}$$

Thus, for $x = 15'$, the substitution of x for $\sin x$ introduces the error

$$\frac{1}{6}\left(\frac{1}{4}\right)^3 \times 1'' = \frac{1''}{384}.$$

For the same value of x, the substitution of 1 for $\cos x$ produces the error

$$\frac{1}{2}\left(\frac{1}{4}\right)^2 \times 1' = \frac{1'}{32} = 2'', \quad \text{approximately}.$$

It will be noted that the use of the suggestion in (4) will frequently make possible substitutions of this character, for an arrangement of the formulae expressing the difference between the required quantity and a nearly equal known quantity oftentimes introduces the trigonometric functions of small angles.

As an illustration, the equation

$$\sin h = \cos \pi \sin \phi + \sin \pi \cos \phi \cos t \tag{15}$$

expresses the relation between the north polar distance of a star, π, its hour angle, t, its altitude h, and the latitude of the place, ϕ. The latitude can be calculated when the remaining quantities are known. One of the most convenient methods results from an application of (15) to the star Polaris. Since the latitude equals the altitude of the celestial pole above the horizon, and since this latter differs at most by 1° 12′ from

the altitude of Polaris, it is desirable to express (15) as a function of $H = \phi - h$. The resulting equation in H is

$$\sin H = -\sin \pi \cos t + \tan \phi \,(\cos H - \cos \pi). \qquad (16)$$

Since $H \lessgtr \pi = 1^\circ\ 12'$, (16) can be replaced by the approximate relation

$$H = -\pi \cos t + \frac{\pi^2}{2} \tan \phi \sin^2 t\,, \qquad (17)$$

in which terms of the order of π^3 or higher have been neglected. The maximum error of (17) is 1″ or 2″. The appearance of ϕ in the last term of (17) seems to require a knowledge of the quantity for whose determination the equation has been designed, viz., the latitude. But, since the coefficient of this term is very small, a rough approximation for ϕ, which can be obtained in a variety of ways, is all that is required.

(e) *Control and Checking of Computations.*

The acquirement of accuracy in the performance of numerical calculations is largely a matter of intelligent practice. The mistakes of the beginner are due in part to an inability to concentrate his attention upon the large number of details involved in even a relatively short calculation. With experience, the more frequently occurring operations are reduced to a more or less mechanical process. This reduces the strain upon the attention, and thereby lessens the liability to error. But this liability is never entirely removed, and the most skillful computer occasionally makes mistakes. It is therefore essential that all calculations should be controlled. The importance of this cannot be too strongly emphasized, especially for the beginner. The methods of checking are numerous. Generally each type of problem requires its own special method of treatment. One of the most satisfactory methods of control is afforded by the derivation of a result from two independent sets of formulæ. There are many cases, however, in which the application of such a test is impossible. Another is the so-called method of differences, which can be applied when the same set of formulæ is to be calculated for several uniformly varying sets of data. In such cases the final results, and indeed, the numerical values of any given quantity in the calculation, present a systematic variation as one passes successively from one set of data to another. The test is applied by forming the successive differences of the numerical values of the final results or of such quantities as it is deemed necessary to

control. A numerical error in one or more of the separate calculations reveals itself through an irregularity in the variation of the differences. The control is a searching one and is capable of bringing to light even the neglected decimals forming a part of the accumulated error of calculation. The arithmetic mean of a series of numbers can be checked by forming the differences between the mean and the individual numbers, regard being paid to the algebraic sign. If the calculated mean is correct, the sum of the positive differences will equal that of the negative differences.

Other methods of checking will readily suggest themselves to the computer after a moderate amount of experience, but whatever the method, he must constantly be on his guard not to place too much confidence in a mere agreement of numerical results, for no process of checking is absolute. An agreement in numerical results signifies at most a certain *probability* that a calculation has been correctly performed. This probability may be large or small according to the nature of the test, but it never becomes equal to certainty. For example, the method of differences affords an invaluable control in so far as isolated errors are concerned, but it is quite incapable of discovering systematic errors affecting similarly all of the separate calculations. It is therefore desirable that several different tests should be applied. For those rare cases in which all ordinary processes of checking become impossible, the calculation should be repeated by a second computer, working quite independently of the first; and failing this, by the original computer himself. But this last method should be adopted only as a final resort, and then only after a considerable interval of time has elapsed, for it is a well-known fact that an error once committed is very likely to recur in the repeated calculation.

The preceding remarks relate particularly to the detection of errors already committed. Something should also be said as to the methods of *preventing* errors. The most important things are care, attention, and deliberation. The systematic arrangement of the work explained under (a) contributes much. The method of combining numbers is of importance. When two numbers are to be added or subtracted, the combination should be made from *left* to *right*. With a little practice this method affords an increase in both accuracy and rapidity as compared with the usual process. With the beginner each result should be verified as the calculation progresses, always

by an independent method, if possible. Thus, additions, in the case of two numbers, can be performed both from left to right and from right to left; for more than two numbers, by adding from the top downward and from the bottom up; subtractions can be controlled by adding the difference to the subtrahend; interpolations of trigonometric functions, by a comparison of the difference of the logarithms of the sine and cosine with the logarithm of the tangent. Finally, the computer should be on the watch for such impossible results as values of a sine or cosine greater than unity, and the occurrence of negative values for essentially positive numbers.

Under no circumstances should these details be neglected unless the computer has acquired a very considerable skill, for it is a matter of experience that the location and correction of errors in a completed calculation consumes far more time than is required for the execution of the work with that deliberation which assures some degree of accuracy in the final result. In addition, the consciousness of having exercised all possible care gives a feeling of security which, in the attempt to secure freedom from errors, is a psychological factor of no small importance.

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AN ASTRONOMICAL THEORY OF THE MOLECULE AND AN ELECTRONIC THEORY OF MATTER.

Solar and Terrestrial Physics Viewed in the Light Thereof.

SEVERINUS J. CORRIGAN.

Part III. (Continued.)

THE NATURE OF THE SOLAR RADIATIONS AND THEIR RELATION TO TERRESTRIAL MAGNETISM AND OTHER METEOROLOGICAL PHENOMENA.

FOR POPULAR ASTRONOMY.

An inspection of the two last columns of Table A clearly shows the predominating influence of the great planet Jupiter in respect to solar radiation and the consequent phenomena, particularly as to the cooling effect upon the Sun's surface matter, which is the cause of these phenomena. The very considerable influence of each of the three smaller planets, Mercury, Venus and the Earth is, also, distinctly indicated, while the quite insignificant values of E in the case of the large outer planets Uranus and Neptune, the considerable, but not great value in that of Saturn and the very small effect

due to Mars, corroborate my claim that only the eight known planets of the solar system need be regarded as sensible factors in the process of solar radiation as viewed in the light of my theory thereof.

As the planets are all revolving around the Sun, their projected areas, and the corresponding regions of *greatest cooling*, upon the solar surface, obviously move, *pari-passu*, in heliographic longitude, and it is evident that there is a considerable number of combinations possible among all these planetary effects, so that at times there may be at least an approximation to *conjoined* action between Mercury, Venus and the Earth —either separately or together—and Jupiter; on the other hand the influences of these three smaller planets may be exerted *against* that of the great planet, the combined influence of the three planets aforesaid being almost equal to that of Jupiter particularly when Mercury is in "perihelion," when its influence is 58½ per cent *greater* than that indicated for mean-distance in Table A, the influence of Jupiter being *less* than the mean indicated, by about ten per cent when that planet is in "aphelion," and greater by about the same amount when it is in "perihelion". If Jupiter were the only planet thus reacting upon the Sun, the aforesaid effects caused thereby would, in all probability, have a period equal to that of this planet's sidereal revolution which is 11.86 years, but by reason of the competitive action of the planets of shorter period, and particularly of these three above referred to, this must be reduced *somewhat* toward Wolf's period of sun-spot variation, which is 11.11 years, or three-quarters of a year less than the period of Jupiter.

The action of the more slowly moving planets, Saturn, Uranus and Neptune, tends to lengthen the period above that of Jupiter, but, as indicated in Table A, their influence in this respect is small. The researches of some of the earlier and eminent investigators in this part of the field of solar physics—De la Rue, Stewart and Loewy—indicated laws of variation connecting spot-frequency with the movements of the planets Jupiter, Earth, Venus and Mercury, and the great specialist in this line, Wolf, accepted the theory of *planetary influence*. He projected the results of his observations of spot-frequency in a continuous curve and found therein a series of small undulations succeeding each other at an average interval of 0.637 of a year, and as the periodic time of Venus is 0.616 of a year, the coincidence is sufficiently close

to warrant a belief in the physical connection aforesaid. A longer period of variation was suspected by Wolf, and a probable connection between this and the conjunction of Saturn with Jupiter was suggested by the late Professor Proctor in 1865 and subsequently advocated by Professor Loomis, the distinguished astronomer and meteorologist,—then of Yale University. Observations of sun-spots made by later investigators tend to confirm these views but, in so far as I have been able to learn, no one has, heretofore, traced this *planetary influence* upon the *frequency* and heliographic distribution of sun-spots, to the correlation between solar thermal radiation and the absorption thereof by the planets in the manner conceived by my theory.

My theoretical investigation as to the distribution of the areas of *greatest cooling* upon the Sun's surface with respect to *heliographic latitude* has led to even more striking and suggestive results, as is indicated by the values of (l') set forth in Table B wherein it appears that the highest limiting latitudes within which such areas are to be found are $\pm 72°.3$, in the case of those due to the planet Neptune, while the lowest are $\pm 5°.0$ in the case of Mercury and these are almost exactly the limiting latitudes—north and south of the solar equator—between which sun-spots—which are here regarded as caused by downfalls of cooler, heavier matter (made so by radiation from the projected planetary areas) through the *photosphere* and into the interior of the solar globe whence this matter has previously arisen in a hotter and, therefore, lighter state are found. Since, as indicated by the respective values of E in Table A, the *cooling area* due to the influence of each of the planets Neptune and Uranus is extremely small, sun-spots caused by these planets at the high latitude, $\pm 72°$ and $\pm 68°$ respectively, must be small and, by reason of the slow motion of these two outer planets, quite infrequent, those due to Saturn (latitude $48°$) being somewhat more notable and frequent—but the Martian effects (latitude $\pm 13°$) are quite inconsiderable. At latitude $\pm 34°$ the predominant action of Jupiter in this process of sun-spot formation begins, followed at latitudes $\pm 7°$ by the considerable effects due to the Earth; at latitude $\pm 6°$ by those attributable to the planet Venus, and at $\pm 5°$ by the disturbance due to Mercury. Of course all these planets, by reason of their revolution around the Sun, exert a spot-producing influence at all latitudes within the limiting ones

aforesaid, from the farthest northern to the uttermost southern limits across the Sun's equator, but any spot at a higher latitude than that given for any planet, in Table B, must be attributed to some one of the planets of greater mean-distance, as is evident from the equation whence these limiting latitudes (l') have been derived. Indeed, from the values of E in Table A and these values of (l') from Table B, together with the magnitude of the disturbed solar area, we may be able to identify any spot with the planet causing it.

There are two facts which account, simply and satisfactorily for the comparative infrequency of sun-spots on, or very near, the solar equator, the first being that the motion of the planets and, therefore, of the areas of maximum cooling effect which they cause upon the Sun's surface and which are factors in the formation of the spots by the descent of the cooled and heavier matter through the hotter and brighter substances constituting the "photosphere," is much greater with respect to heliographic latitude, when the planets are near the nodes of their orbits, with the plane of the solar equator, than when they are in the vicinity of 90° from said nodes, just as the apparent motion of the Sun, in "declination" is much greater at, and near, the "equinoxes," than at, or near, the "solstitial" points, 90° distant, the time of passage across the equatorial belt being, therefore, comparatively short and the opportunity for spot-formation correspondingly less than in the higher solar latitudes where sun-spots are generally observed. The second fact is that, while maximum areas of cooling do exist in the regions near the solar equator and even on that line when the planets causing them are crossing it and the cooled denser matter tends to sink downward under the *centripetal* action of solar gravity, there is in operation also, a *centrifugal* force and a component *tangential* motion, due to the Sun's axial rotation, which is greatest at the solar equator, this factor operating to prevent or, at least, to retard, the fall of this matter and to cause it to move forward in the direction of the rotation from east to west, upon the Sun's disk, this action, obviously tending to greatly lessen the probability of the formation of sun-spots in the solar equatorial regions although they do occasionally appear there; the greater thickness of the solar atmosphere near the equator also operates to retard the fall. By reason of temperature differences, there must be horizontal convection-currents, along the Sun's surface, from both the polar and the equatorial regions,

toward the zones of sun-spots, or the areas of greatest cooling, on each side of the solar equator, which movement, together with that caused by the Sun's rotation, must result in a tendency to vortex—motion in the spotted areas—"clockwise" in the southern, and "contra-clockwise" in the northern hemisphere of the Sun, in a manner analogous to that in the case of cyclonic movements in the terrestrial atmosphere. Furthermore, the hotter matter ejected from the interior and cooled by radiation, in the upper portions of the solar atmosphere (appearing therein as the faculæ) and becoming, therefore, denser, may well have a surface drift of as much as 10°—or even more—both in heliographic latitude and longitude, before descending to, and penetrating, the photosphere, thereby causing the appearance of a "sun-spot". Thus, in the case of Mercury, Venus and the Earth, the limiting latitudes of the "projections" of the planetary areas are ± 5°; ± 6° and ± 7°, respectively, so that while these may mark the extreme regions of the areas of greatest cooling due to these planets, the solar matter, so cooled, may *drift* northward and southward, to latitudes 15°, or 20°, before falling to a depth sufficiently great to cause the appearance of a "sun-spot"; and, for the same reason, the matter cooled in latitude ± 34°, by radiation to the planet Jupiter may *drift* down to ± 25° or even lower, before penetrating the photosphere, and the same reasoning applies in the case of the planet Saturn. Furthermore, since by the rotation of the solar globe at a greater *angular* rate than that of the revolution of the planets, in their orbits in the same direction, the projection of the planetary areas must appear to lag behind in heliographic longitude and, therefore, the corresponding areas of maximum cooling, due to each planet, must extend in elongated belts parallel to the solar equator, and the resultant sun-spots must often, on this account, appear in groups, or "chains," extended in an easterly and westerly direction on the Sun's disk. The drifting of the "faculae" and the "spots", as well as the grouping of the former about and above the latter, and the formation of groups of "spots" extending across the disk, in lines roughly parallel to the solar equator, are facts well attested by observation all constituting testimony, of no inconsiderable weight, in favor of my theory in this connection.

These areas of greatest cooling effect upon the Sun's surface are, of course, not limited, strictly, to the *projections* of the areas of the planets causing them, but may extend over several

degrees of heliographic latitude and longitude, but it is quite reasonable to assume that the mean diameter of a sun-spot, or disturbed area, will bear some comparatively close proportion to the diameter of the planet causing the disturbance in the manner aforesaid, and we may properly take the ratios anywhere between one-fourth and four times the planetary diameter, as the limiting values of the mean diameters of the said "spots," or areas. Thus in the case of disturbances caused by the planet Mercury, this diameter would be from 750 miles to 12,000 miles, corresponding to an angular diameter on the solar disk of $2'' - 24''$; in the case of the planet Venus, and also in that of the Earth, the value may lie between 2,000 miles and 32,000 miles ($5'' - 72''$), and in that of Jupiter and also Saturn between 20,000 miles and 320,000 miles ($45'' - 720''$) all of which theoretical values agree well with those observed, there being of course occasional combinations of disturbed areas due to approximate conjunctions of the planets—particularly Mercury, Venus and the Earth each of which is of comparatively short "period."

When the Earth is at 90° from the nodes of the planets, with reference to the solar equator, the plane of its orbit approaches most nearly to those of the other planets when they are in such position that their action upon the Sun is most intense, so that whatever *reaction* upon the Earth and its atmospheric envelope is caused by the outbursts of the more highly heated matter from below the surface of the solar globe should, likewise, be of maximum intensity at, or about, the times when the Earth is in the aforesaid position with respect to each of the other planets—the dates of this event, in each case, being set forth in the last column of Table B. These outbursts of superheated gases are caused by the violent descent of the superficial matter of the Sun, made cooler and denser by radiation to each planet, this matter immediately upon cooling, and before descending, appearing as the "faculae" which are observed to hover, in greatest number, in the vicinity of and even over the "spots," while the displaced and ascending ejected gases from the interior appear in the form of the red "prominences," "protuberances," "flames" "eruptions" and "jets," the two last phenomena being especially results of the more sudden and heavy downfalls of the cooler masses which displace portions of the hotter internal matter and cause them to be violently ejected upward, along the lines of least resistance, this matter appearing in the form of the

"jets," "eruptions," etc., somewhere in the vicinity of the "spots", the slower, and more uniform processes of convection giving rise to the quiet "prominences," "domes" and kindred phenomena observed above the Sun's photosphere. Now, it is obvious that if the effects caused, at the Earth, by these solar disturbances, be due *directly* to the sun-spots, such effects should be most marked when the "spots" are upon or near the central meridian of the solar globe, and when the vertical axis of the spot is directed most nearly toward the Earth, but if the terrestrial meteorological phenomena—aurorae, magnetic storms and perturbations, areas of low barometric pressure and consequent storm centers—be due (as I claim that they are) to an abnormal radiation consequent upon the ejection of hotter matter from the interior of the Sun, then these phenomena must be manifested *subsequently* to the appearance of the sun-spot, because it is very evident that the "jets," "eruptions," etc., must *follow*, in point of time, the downfall of the cooled masses, through the photosphere, (whereby the "spots" are caused) the outbursts of superheated matter, and their consequent effects occurring after the lapse of a sensible, and even a considerable, time after the transit of a sun-spot over the "central meridian."

Moreover, since the intensity of the divers meteorological disturbances depends upon, both the intensity and extent of the eruptive action upon the Sun, and the direction of the vertical axes of the "jets" etc., with respect to the Earth, it follows that, in the case of many notable "spots" in which the *downfall* of the cooled masses is gradual and not violent, or the vertical axis of the ejected mass is directed away from the Earth, there may be no sensible meteorological disturbances whatever when a large sun-spot transits the "central meridian" of the solar globe, and since these "jets" and "eruptions" may be active–particularly in the equatorial regions—without the appearance of a sun-spot, magnetic and other meteorological phenomena may be manifested, as they often are, in the *absence* of the "spots"—all of which confirms my claim that it is the ascending ejected hotter matter from the interior of the Sun (appearing as the "jets and eruptions") and not the cooled descending matter (productive of the "spots,") that, by its excessive radiation, is the chief factor in the causation of the meteorological phenomena displayed at the Earth's surface and in the atmospheric envelope of our globe.

I shall now introduce some testimony based upon observational data, in support of my views on the several points above discussed in this part of my paper. In a communication published in No. 111 of ASTRONOMY AND ASTRO-PHYSICS (Jan. 1893) under the title "Sun-spots and Magnetic Perturbations in 1892," Professor A. Ricco of the Observatory of Catania, Sicily, gave the results of his observations of certain sun-spots very prominent at about the time of the "maximum" just preceding the last one, recently passed, and he correlated them therein, with certain data derived from the curves of the photo-magnetographs for 1892, which had been sent to him by the United States Naval Observatory; from the data in his communication I have made the following tabulation:

Table C.

No.	Date 1892	Principal Sun-spot	Diameter (Miles)	Heliographic Latitude	Magnetic Perturbations	Deviation	Retardation	Disturbing Planets
				°		′	Hours	
1	Jan. 4	Very Large	32,000	+ 20	Very Large	42	38	Saturn; Mercury; Earth
2	" 28	Large	24,000	— 16	Large	28	21	Jupiter; Venus
3	Feb. 12	Extraordinary	88,000	— 30	Extraordinary	90	45	" "
4	Mar. 7	Near East Limb	50,000	— 29	Large	48	48	Jupiter:Mercury
5	" 10	Extraordinary	48,000	— 29	Very Large	53	45	" "
6	April 23	Large	20,000	+ 11	Large	41	51	Venus; Mercury Earth
7	" 24	Large	24,000	+ 16	Large	46	45	Venus; Mercury Earth
8	May 16	Extraordinary	40,000	— 16	Extraordinary	42	49	Jupiter

The transits of the "spots" across the central meridian of the Sun occurred between 2:00 p. m. and 8:00 p. m. in all cases except those of Nos. 3 and 4, in which they took place between 2:00 a. m. and 4:00 a. m.

An inspection of Table C discloses, at once, the very significant facts that the dimensions of the principal sun-spots, and the extent of the magnetic-perturbations as measured by the maximum "deviation" of the magnetic needle from its undisturbed position in "declination", as set forth (in minutes of arc) in the seventh column, *correspond exactly*, and that the time which elapsed between the transit of each sun-spot across the "central meridian" of the solar globe, and the occurrence of the respective maximum perturbation of the magnetic-declinations, as set forth in the eighth column, was between 21 hours and 51 hours, the "retardation" being therefore, on an average, 36 hours, which fact tends strongly to corroborate my claim that the magnetic perturbations are caused by ex-

cessive—but short-lived—radiation from matter ejected from the interior of the Sun (maximum internal absolute temperature 13,400° F) heated above the mean, normal radiating temperature of the Sun's surface matter, which is roundly 6,700° Fahrenheit, according to my several determinations referred to in preceding parts. This latter is practically the absolute temperature between the poles of an arc-lamp operating in atmospheric air at normal pressure, the atmospheric molecules between said "poles" being *disrupted* by the passage of the electric current, and the component atoms of these molecules being continually subjected to a process of "dissociation" and "recombination" while the lamp is in operation. It is just above the absolute temperature (6679° Fahrenheit) at, and above, which, according to a fundamental concept of my "electronic theory of matter"—the component atoms of each molecule of a gas at normal atmospheric pressure and temperature, grouped two and two, in myriads of "diatomic-couples" in each molecule, are "dissociated," the molecules composed thereof *disrupted* and their atoms released from the closed paths around each molecular center around which (according to my "astronomical theory of the molecule") they have been revolving, under Kepler's 3rd law, in normally *circular* orbits when undisturbed by impacts from the vibrating atoms of a source of heat, or radiating surface; in *elliptical* orbits the eccentricity whereof increases with the vibrational velocity of said atoms (which velocity is proportional to the absolute temperature of the radiating surface) when affected by *radiation*, and in *hyperbolic* orbits when the orbital velocity of the revolving atoms of the gaseous matter has been increased (by the velocity imparted by the impinging atoms of the heated surface) beyond the *parabolic* limit which according to my determination, to be set forth subsequently, corresponds to the absolute temperature 6679° Fahrenheit in the case of radiation in atmospheric air at normal pressure and temperature. When this limiting temperature, and corresponding orbital velocity, is exceeded the component atoms of each gaseous molecule affected by the radiation, fly off in practically straight lines, this action constituting an electrical discharge, according to a fundamental concept of my theory.

The names of the planets active in the formation of the respective sun-spots are set forth in the last column of Table C. I having derived these data from the heliocentric positions of planets as given in the *American Ephemeris* for 1892 and from

the values Ω', i' and l' in Table B in May number of POPULAR ASTRONOMY. It should be noted here that, by reason of a clerical error in copying from my computing sheets, the longitude (Ω') of the ascending node, the inclination (i') and the limiting heliographic latitudes (l'), with respect to the solar equator, should be, in the case of Jupiter, 84°.8, 7°.0 and 32°.4 and in that of Saturn 92°.2, 6°.6 and 47°.6, instead of the values of these quantities set forth in Table B, and also that in connection with the prominent sun-spots here discussed the action of other planets Mars, Uranus and Neptune may be ignored because, as indicated by the values of E in Table A, this action is comparatively very small in each case, so that the discussion will be confined to the planets named in Table C, above. I have found that on January 4, 1892, Saturn, Mercury and the Earth were the only planets "projected" against the northern hemisphere of the Sun, and in a resultant position such as would cause a sun-spot in heliographic latitude + 20°, having regard to the *drifting* of the cooled matter in the upper regions of the solar atmosphere, southward from the "projected" latitude of the planet Saturn, and northward from that of each of the two other planets, the names of all these bodies being set forth in the order of the intensity of their action. The diameter of this sun-spot as set forth in Table C, was 32,000 miles which is nearly one-half that of the principal planet—Saturn—active in the causation of the "spot," both Saturn and Mercury being then "projected" against the visible hemisphere of the Sun. In the case of the sun-spot crossing the "central meridian," on January 28, 1892 in heliographic latitude — 16°, the only planets in position to effect this result were Jupiter and Venus, the diameter of the principal "spot" being only a little less than one-third the diameter of Jupiter which was projected against the invisible hemisphere of the Sun, to the south of the solar equator, while the projection of Venus was on the visible hemisphere and, likewise, to the south. The principal sun-spot crossing the "central meridian," on February 12, was due, in greatest part, to the action of Jupiter which was then projected against the invisible hemisphere of the Sun almost exactly in heliographic latitude — 30°, this "spot," therefore, having originated on the other side of the solar disk, appearing, thereafter on the eastern "limb," while Venus, projected against the visible portion of the disk, acted in a subsidiary manner. The diameter of the principal "spot" in this case was 88,000 miles, or about

equal to the combined diameters of the two planets, which fact taken in connection with their positions relative to the Sun, makes it quite evident that Jupiter and Venus—particularly the former—were the only factors in the causation of this sun-spot. On March 7, as the record shows, a similar, large "spot" was observed near the eastern limb of the Sun, and on March 10 it crossed the "central meridian," in heliographic latitude —29°, or within one degree of the position of the "spot" observed on February 12, the principal factor in these two latter cases being also the planet Jupiter, with Mercury acting in a subsidiary manner, the diameters of these "spots" being about one-half the combined diameters of the planets causing them. The position of the chief factor Jupiter, on March 7th and 10th, was on the side of the Sun opposite to that visible from the Earth, so that, as in the case on February 12th, the "spot" must have been formed on the invisible hemisphere of the solar globe and brought into view, around the eastern limb, by the axial rotation of the Sun. The "spot" observed on March 7th and also that on March 10th, were regarded by the observer as a return of the "spot" of February 12, and it may have been, but, under my theory of the causation of these phenomena, it may with equal probability have been one *newly formed* on the opposite side of the Sun, during the interval between February 12th and March 7-10, and this raises a question as to the time of duration of a "spot," and casts some doubt as to the alleged persistence of this phenomenon through one or more complete rotation periods of the Sun. It is quite evident that, under my view, a sun-spot newly formed by a planet of "long period"—such as Jupiter or Saturn —would be situate in nearly the same heliographic latitudes as, and present a somewhat similar appearance to, one formed by either planet on the opposite side of the Sun during one, or more, immediately preceding rotation periods of that body. The diameter of the "spots" of March 7-10 was 42,000 miles, or about one-half the combined diameters of the planets causing them.

The sun-spots that crossed the "central meridian" on April 23 and 24, in latitudes + 11° and + 16°, were caused by Venus, Mercury and the Earth, the diameters, which were respectively 20,000 miles and 24,000 miles, being about equal to the combined diameters of these three planets, and nearly three times that of Venus, or the Earth, considered separately, and, on each date, Venus and Mercury were projected against the visible hemisphere of the Sun.

The sun-spot that crosssd the "central meridian" on May 16th, in latitude — 16° was due practically to Jupiter alone, its diameter having been about one-half that of said planet which was, on that date, projected against the southern hemisphere of the Sun, sufficiently near the heliographic latitude at which the "spot" appeared. On a preceding page I stated that, according to my theory, there must be some near proportion between the diameter of a sun-spot and that of the planet, or planets, causing it, and placed the ratio at roughly from ¼ to 4, but, from the records above discussed, it appears that the limiting ratios were much less than these, being more nearly ½ to 2, which fact is strongly corroborative of my theory in this connection. As stated, the relation between the magnitude of each sun-spot included in Table C, and the range of the corresponding magnetic perturbation at the Earth, is most remarkable, while the "retardation" of these "perturbations" is in strict accordance with my theory as to the occurrence of "eruptions" and "jets" of very highly heated matter from just below the solar photosphere, as a *consequence* of the downfall of the cooled heavier matter from the upper regions of the solar atmosphere, productive of the "spots". Perturbations, or fluctuations of the three elements of "terrestrial magnetism" are very evidently caused by actions taking place upon the Sun, the "diurnal," "monthly," "semi-annual" and "secular" irregularities of magnetic declination-range and magnetic storms, being analogous to irregularities of atmospheric "temperature," "pressure" and correlated "humidity," which depend upon solar action, and are recognized as well established factors in the causation of the combination of ordinary meteorological phenomena which we call "weather"—there being also what may be termed "magnetic-weather". Therefore a study, from the view-point of my theory of the nature and *modus operandi* of the electro-magnetic, and other radiations from the Sun, by and through the luminiferous ether, and of the molecular constitution and extent of the terrestrial atmosphere through which these radiations are transmitted to the Earth's surface after undergoing certain important modifications dependent upon the molecular properties of the atmospheric gases, may throw some light upon the nature of the relations between these solar radiations and "terrestrial magnetism", as well as other meteorological phenomena, which relation—not long ago wholly unknown, or doubted—has during the past decade, or two, become more, and more, apparent to physicists, although the nature thereof has not heretofore been definitely ascertained.

Saint Paul, Minnesota.

To be continued.

PLANET NOTES FOR JULY AND AUGUST 1908.

H. C. WILSON.

Mercury will be at inferior conjunction July 4, and will be visible as morning star during the last two days of the month. The planet will be at greatest elongation, 19° 51′ west from the Sun on July 25. It will not be

THE CONSTELLATIONS AT 9:00 P. M., AUGUST 1, 1908.

visible during August, being at superior conjunction August 20. Mercury will be in conjunction with Neptune, 0° 44′ south of the latter, on the morning of July 28. On August 18 at 12^h C. S. T., Mercury will be 1° 2′ north of Jupiter and on August 20 at 2^h C. S. T., it will be 40′ north of Mars.

Venus will be at inferior conjunction on July 5, and so will not be visible during the first half of the month. After that she will be seen as morning star, reaching her greatest brilliancy August 11 and then waning slowly for several months. Venus and Mercury will be in conjunction a little way west of the Sun on the morning of July 15.

Mars will be at conjunction with the Sun on August 21 and will be too close to the Sun to be observed during the summer. Mars will be in conjunction with Jupiter on August 13 and with Mercury August 20.

Jupiter will be in conjunction with the Sun August 17 and so may not be observed during the summer.

Saturn is at quadrature 90° west from the Sun July 1 and will be visible in the latter half of the night. The plane of the rings now makes an angle of nearly eight degrees with the line of sight from the Earth, so that th rings can be fairly well seen. Saturn is to be found easily without the aid of a telescope, toward the southeast in the morning, in the constellation Pegasus.

Uranus will be at opposition July 6 and so will be in best position for study during the summer months. This planet requires the use of a good telescope, and may be found in the constellation Sagittarius.

Neptune will be in conjunction with the Sun July 6 and so cannot be seen during the summer.

Occultations visible at Washington.

Date 1908	Star's Name	Magnitude.	IMMERSION. Washington M.T.	IMMERSION. Angle f'm N.	EMERSION. Washington M.T.	EMERSION. Angle f'm N	Duration
			h m	°	h m	°	h m
July 4	ν Virginis	4.2	9 27	67	10 08	346	0 41
12	26 Sagittarii	6.1	6 42	32	7 06	348	0 24
14	27 Capricorni	6.1	15 15	88	16 19	228	1 04
17	30 Piscium	4.7	14 24	338	14 34	322	0 10
17	33 Piscium	4.7	15 57	42	17 12	251	1 15
18	20 Ceti	4.9	12 35	117	13 13	187	0 38
Aug. 16	n Virginis	6.5	10 01	99	11 01	304	1 00
5	o^1 Libræ	6.2	7 00	114	8 24	297	1 24
8	Bradley 2276	5.2	5 52	52	04 38	334	0 46
9	B A.C. 6576	6.1	5 39	106	6 43	266	1 06
9	χ^3 Sagittarii	5.5	10 51	49	11 56	296	1 05
10	Piazzi xx, 146	6.2	9 57	59	11 10	278	1 13

COMET NOTES.

Ephemeris of Comet 1907 *d*.

(For Berlin Midnight, from *A. N.* 4245.)

1908	α 1908.0 h m s	δ 1908.0 ° ′	log r	log Δ	Mag.
June 1	13 52 06	−0 14.3	0.6193	0.5303	11.7
3	51 16	0 14.2			
5	50 30	0 14.6	0.6239	0.5416	
7	49 48	0 15.4			
9	49 10	0 16.7	0.6285	0.5530	11.9
11	48 36	0 18.3			
13	48 06	0 20.2	0.6330	0.5644	
15	47 40	0 22 5			
17	47 17	0 25.2	0.6374	0.5756	12.1
19	46 59	0 28.2			
21	13 46 44	−0 31.5	0.6417	0.5869	

Ephemeris of Comet 1907 *d*.—Continued.

1908	α 1908.0 h m s	δ 1908.0 ° ′	log *r*	log Δ	Mag.
June 23	13 46 33	−0 35.2			
25	46 26	0 39.1	0.6460	0.5981	12.2
27	46 22	0 43.3			
29	46 21	0 47.8	0.6502	0.6091	
July 1	46 23	0 52.5			
3	46 29	0 57.5	0.6544	0.6200	12.4
5	46 38	1 02.7			
7	46 50	1 08.1	0.6585	0.6307	
9	47 05	1 13.7			
11	13 47 24	−1 19.6	0.6626	0.6412	12.5

VARIABLE STARS.

Minima of Variable Stars of the Algol Type.

[Given to the nearest hour in Greenwich Mean Time beginning with noon. To reduce to Central Standard time subtract 6 hours, or for Eastern time subtract 5 hours. Alternate minima are given this month.]

U Cephei

	d	h
July	3	2
	8	1
	13	1
	18	1
	23	0
Aug.	2	0
	6	23
	11	23
	16	23
	21	22
	26	22
	31	22

Z Persei

July	1	18
	7	21
	14	0
	20	3
	26	5
Aug.	1	8
	7	11
	13	13
	19	16
	25	19
	31	21

RY Persei

July	4	12
	18	6
	31	23
Aug.	14	17
	28	10

RZ Cassiop.

July	2	4
	4	13
	6	23
	9	8
	11	18

RZ Cassiop.

	d	h
July	14	3
	16	12
	18	23
	21	7
	23	16
	26	1
	28	11
	30	21
Aug.	2	6
	4	15
	7	1
	9	11
	11	19
	14	5
	16	14
	18	23
	21	9
	23	18
	26	4
	28	13
	31	22

RX Cephei

July	2	4
Aug.	3	12

Algol

July	3	19
	9	13
	15	7
	21	0
	26	18
Aug.	1	11
	7	5
	13	0
	18	16
	24	10
	30	4

RT Persei

	d	h
July	1	3
	2	20
	4	12
	6	5
	7	22
	9	15
	11	7
	13	0
	14	17
	16	10
	18	3
	19	19
	21	12
	23	5
	24	22
	26	14
	28	7
	30	0
	31	17
Aug.	2	9
	4	2
	5	19
	7	12
	9	5
	10	21
	12	14
	14	7
	16	0
	17	16
	19	9
	21	2
	22	19
	24	12
	26	4
	27	21
	29	14
	31	7

λ Tauri

	d	h
July	4	0
	11	22
	19	19
	27	17
Aug.	4	15
	12	13
	20	10
	28	8

RW Tauri

July	3	0
	8	13
	14	2
	19	15
	25	4
	30	16
Aug.	5	5
	10	18
	16	7
	21	20
	27	9

RV Persei

July	1	7
	5	6
	9	5
	13	4
	17	2
	21	1
	25	0
	28	23
Aug.	1	21
	5	20
	9	19
	13	17
	17	16
	21	15
	25	14
	29	12

RW Persei

	d	h
July	7	16
Aug.	3	1
	29	11

RS Cephei

July	11	2
Aug.	4	22
	29	18

RW Geminorum

July	2	0
	7	18
	13	11
	19	5
	24	22
	30	16
Aug.	5	9
	11	3
	16	21
	22	14
	28	8

Y Camelop.

July	1	10
	8	1
	14	15
	21	6
	27	21
Aug.	3	12
	10	2
	16	17
	23	7
	29	22

SS Carinae

July	1	13
	8	4
	14	8
	21	9
	27	23

Minima of Variable Stars of the Algol Type.—Continued.

SS Carinæ		RZ Centauri		U Ophiuchi		V Serpentis		SX Sagittarii	
d	h	d	h	d	h	d	h	d	h
Aug. 3	13	Aug. 27	1	July 17	0	July 2	6	Aug. 24	21
10	4	28	22	18	16	9	4	29	1
16	18	30	19	20	8	16	2	RR Draconis	
23	9	SS Centauri		22	0	23	0	July 2	3
29	23	July 2	13	23	17	29	21	7	19
Z Draconis		7	12	25	9	Aug. 5	19	13	11
July 2	7	12	11	27	1	12	17	19	3
5	0	17	10	28	17	19	15	24	19
7	17	22	9	30	10	26	12	30	11
10	10	27	8	Aug. 1	2	RX Herculis		Aug. 5	3
13	4	Aug. 1	7	2	18	July 1	14	10	19
15	21	6	6	4	10	3	8	16	10
18	14	11	5	6	3	5	3	22	2
21	7	16	4	7	19	6	22	27	18
24	0	21	3	9	11	8	16	U Scuti	
26	17	26	2	11	3	10	11	July 1	1
29	10	31	1	12	20	12	6	2	23
Aug. 1	4	δ Libræ		14	12	14	1	4	21
3	21	July 2	12	16	4	15	19	6	19
6	14	7	4	17	21	17	14	8	16
9	7	11	19	19	13	19	9	10	14
12	0	15	11	21	5	21	3	12	12
14	17	21	3	22	21	22	22	14	10
17	10	25	19	24	14	24	17	16	8
20	4	30	10	26	6	26	11	18	6
22	21	Aug. 4	2	27	22	28	6	20	3
25	14	8	18	29	14	30	1	22	1
28	7	13	9	31	7	31	19	23	23
31	0	18	1	Z Herculis		Aug. 2	14	25	1
RZ Centauri		22	17	July 2	5	4	9	27	19
July 1	18	27	8	6	5	6	3	29	17
3	15	U Coronæ		10	5	7	22	31	14
5	12	July 2	9	14	5	9	17	Aug. 2	12
7	9	9	7	18	4	11	11	4	10
9	6	16	5	22	4	13	6	6	8
11	3	23	3	26	4	15	1	8	6
13	0	30	0	30	4	16	20	10	4
14	21	Aug. 5	22	Aug. 3	4	18	14	12	1
16	18	12	20	7	4	20	9	13	23
18	15	19	17	11	3	22	4	15	21
20	12	26	15	15	3	23	22	17	19
22	9	R Aræ		19	3	25	17	19	17
24	6	July 4	7	23	3	27	12	21	15
26	4	13	4	27	3	29	6	23	13
28	1	22	0	31	3	31	1	25	10
29	22	30	20	RS Sagittarii		SX Sagittarii		27	8
31	19	Aug. 8	17	July 1	20	July 1	21	29	6
Aug. 2	16	17	13	6	16	6	1	31	4
4	13	26	10	11	12	10	5	RX Draconis	
6	10	U Ophiuchi		16	8	14	8	July 2	13
8	7	July 1	21	21	4	18	12	6	8
10	4	3	14	26	0	22	16	10	2
12	1	5	6	30	20	26	20	13	21
13	22	6	22	Aug. 4	16	30	23	17	16
15	19	8	14	9	12	Aug. 4	3	21	11
17	16	10	7	14	8	8	7	25	6
19	13	11	23	19	4	12	10	29	1
21	10	13	15	24	0	16	14	Aug. 1	20
23	7	15	7	28	20	20	18	5	15
25	4							9	10

Minima of Variable Stars of the Algol Type.—Continued.

RX Draconis

	d	h
Aug.	13	5
	17	0
	20	18
	24	13
	28	8

RV Lyræ

	d	h
July	4	3
	11	8
	18	12
	25	17
Aug.	1	22
	9	3
	16	7
	23	12
	30	17

U Sagittarii

	d	h
July	1	7
	8	2
	14	20
	21	14
	28	8

U Sagittarii

	d	h
Aug.	4	3
	10	21
	17	15
	24	10
	31	4

SY Cygni

	d	h
July	5	9
	17	9
	29	9
Aug.	10	10
	22	10

WW Cygni

	d	h
July	2	22
	9	13
	16	5
	22	20
	29	11
Aug.	5	2
	11	18
	18	9
	25	0

WW Cygni

	d	h
Aug.	31	15

SW Cygni

	d	h
July	2	7
	11	10
	20	14
	29	17
Aug.	7	21
	17	0
	26	4

VW Cygni

	d	h
July	4	11
	21	8
Aug.	7	4
	24	1

UW Cygni

	d	h
July	3	22
	10	19
	17	17
	24	15
	31	12
Aug.	7	10
	14	8

UW Cygni

	d	h
Aug.	21	5
	28	3

W Delphini

	d	h
July	2	20
	12	11
	22	1
	31	16
Aug.	10	7
	19	22
	29	12

RR Delphini

	d	h
July	5	5
	14	10
	23	14
Aug.	1	19
	11	0
	20	5
	29	10

VV Cygni

	d	h
July	1	7
	4	6

VV Cygni

	d	h
July	7	5
	10	4
	13	3
	16	2
	19	1
	22	0
	24	23
	27	22
	30	20
Aug.	2	19
	5	18
	8	17
	11	16
	14	15
	17	14
	20	13
	23	12
	26	11
	29	9

UZ Cygni

	d	h
July	31	2
Aug.	31	9

Maxima of Variable Stars of Short Period not of the Algol Type.

Unless otherwise indicated the times of maxima only are given; and the times of minima may be found by subtracting the interval printed in parentheses under the names of the stars.

RW Cassiop.

	d	h
	(−5	19)
July	5	2
	19	22
Aug.	3	17
	18	12

RX Aurigæ

	(−4	0)
July	6	12
	18	4
	29	19
Aug.	10	10
	22	1

Y Aurigæ

	(−0	18)
July	2	19
	6	16
	10	12
	14	9
	18	6
	22	2
	25	23
	29	19
Aug.	2	16
	5	13
	10	9

Y Aurigæ

	d	h
Aug.	14	6
	18	3
	21	23
	25	20
	29	16

RU Camelop.

	(−9	12)
July	9	5
	31	12
Aug.	22	18

S Muscæ

	(−3	11)
July	6	2
	15	18
	25	9
Aug.	4	1
	13	17
	23	9

T Crucis

	(−2	2)
July	2	3
	8	21
	15	14
	22	8
	29	1

T Crucis

	d	h
Aug.	4	19
	11	13
	18	6
	25	0
	31	17

R Crucis

	(−1	10)
July	3	5
	9	1
	14	21
	20	16
	26	12
Aug.	1	8
	7	4
	13	0
	18	19
	24	15
	30	11

S Crucis

	(−1	12)
July	3	22
	8	15
	13	8
	18	0
	22	17
	27	9

S Crucis

	d	h
Aug.	1	2
	5	18
	15	3
	19	20
	24	13
	29	9

W Virginis

	(−8	5)
July	8	9
	25	15
Aug.	11	22
	29	4

V Centauri

	(−1	11)
July	6	14
	12	2
	17	14
	23	2
	28	14
Aug.	3	2
	8	13
	14	1
	19	13
	25	1
	30	13

R Triang. Austr.

	(−1	0)
July	3	9
	6	18
	10	3
	13	13
	16	22
	20	7
	23	17
	27	2
	30	11
Aug.	2	21
	6	6
	9	15
	13	1
	16	10
	19	19
	23	5
	26	14
	29	23

S Triang. Austr.

	(−2	2)
July	5	1
	11	9
	17	17
	24	0
	30	8

Maxima of Variable Stars of Short Period not of the Algol Type Continued.

Aug.	5	16
	d	h
	12	0
	18	8
	24	15
	30	23

S Normæ

	(−4	10)
July	7	18
	17	12
	27	6
Aug.	6	0
	15	18
	26	12

RV Scorpii

	(−1	10)
July	6	21
	12	23
	19	0
	25	2
	31	3
Aug.	6	5
	12	6
	18	8
	25	9
	30	11

RV Ophiuchi Minimum.

July	4	13
	8	6
	11	22
	15	15
	19	7
	23	0
	26	16
	30	9
Aug.	3	1
	6	18
	10	10
	14	3
	17	19
	21	12
	25	4
	28	21

Y Ophiuchi

	(−6	5)
July	4	18
	21	21
Aug.	8	0
	25	3

W Sagittarii

	(−3	0)
July	6	19
	14	9
	21	23
	29	14
Aug.	6	4
	13	18
	21	8
	28	23

Y Sagittarii

	(−2	2)
July	2	2

Y Sagittarii

	d	h
	7	20
	13	15
	19	9
	25	4
	30	23
Aug.	5	17
	11	12
	17	6
	23	1
	28	19

U Sagittarii

	(−2	23)
July	2	9
	9	3
	15	21
	29	9
Aug.	5	3
	11	20
	18	14
	25	8

β Lyræ

	(−3	7)
	(−3	2)
July	7	12
	13	18
	20	10
	26	16
Aug.	2	8
	8	14
	15	6
	21	12

κ Pavonis

	(−1	7)
July	6	16
	15	18
	24	20
Aug.	2	22
	12	1
	21	3
	30	5

U Aquilæ

	(−2	4)
July	5	8
	12	9
	19	9
	26	10
Aug.	2	10
	9	11
	16	12
	23	12
	30	13

U Vulpeculæ

	(−2	3)
July	3	11
	11	11
	19	10
	27	10
Aug.	4	9
	12	9
	20	8
	28	8

SU Cygni

	d	h
	(−1	7)
July	2	8
	6	5
	10	1
	13	21
	17	18
	21	14
	25	10
	29	6
Aug.	2	3
	5	23
	9	19
	13	16
	17	12
	21	8
	25	5
	29	1

η Aquilæ

	(−2	6)
July	5	8
	12	13
	19	17
	26	21
Aug.	3	1
	10	6
	17	10
	24	14
	31	18

S Sagittae

	(−3	10)
July	6	12
	14	21
	23	6
	31	16
Aug.	9	1
	17	10
	25	19

X Vulpeculæ

	(−2	1)
July	4	14
	10	22
	17	6
	23	13
	29	21
Aug.	5	4
	11	12
	17	20
	24	3
	30	11

V Vulpeculae Minimum

July	26	18

X Cygni

	(−6	19)
July	14	21
	31	6
	16	15

T Vulpeculae

	(−1	10)
July	3	4
	7	15
	12	1

T Vulpeculae

	d	h
July	16	12
	20	22
	25	8
	29	19
Aug.	3	5
	7	16
	12	2
	16	13
	20	23
	25	10
	29	20

TX Cygni

July	9	16
	24	9
Aug.	8	2
	22	20

VY Cygni

	(−2	14)
July	8	10
	16	7
	24	3
Aug.	1	0
	8	21
	16	17
	24	14

VZ Cygni

	(−2	12)
	(−3	6)
July	7	2
	12	2
	16	19
	21	19
	21	19
	26	13
	31	13
Aug.	5	6
	10	6
	14	23
	19	23
	24	17
	29	17

δ Cephei

	(−1	10)
July	2	10
	7	19
	13	3
	18	12
	23	21
	29	6
Aug.	3	15
	8	23
	14	8
	19	17

δ Cephei

	d	h
	25	2
	30	11

V Lacertae

	(−0	17)
July	3	22
	8	22
	13	21

V Lacertae

	d	h
July	18	21
	23	21
	28	20
Aug.	2	20
	7	19
	12	19
	17	19
	22	18
	27	18

X Lacertae Minimum

July	2	11
	7	21
	13	8
	18	18
	24	5
	29	15
Aug.	4	2
	9	12
	14	23
	20	10
	25	20
	31	7

RS Cassiop.

	(−1	19)
July	2	21
	9	4
	15	12
	21	19
	28	2
Aug.	3	9
	9	16
	15	23
	22	6
	28	13

RY Cassiop.

	(−7	10)
July	8	14
	20	17
Aug.	1	21
	14	0
	26	4

Y Lacertae

	(−1	10)
July	2	1
	6	8
	10	16
	14	23
	19	7
	23	14
	27	22
Aug.	1	6
	5	13
	9	21
	14	4
	18	12
	22	19
	27	3
	31	11

Approximate Magnitudes of Variable Stars on May. 1, 1908.

[Communicated by the Director of Harvard College Observatory, Cambridge, Mass.]

Name.	R. A. 1900. h m	Decl. 1900. ° ′	Magn.	Name.	R.A. 1900. h m	Decl. 1900. ° ′	Magn.
T Androm.	0 17.2	+26 26	11.0*d*	S Gemin.	7 37.0	+23 41	10.0 *i*
T Cassiop.	17.8	+55 14	12.0 *i*	T Gemin.	43.3	+23 59	12.0*d*
R Androm.	18.8	+38 1	11.0 *i*	U Puppis	56.1	−12 34	14.0
Y Cephei	31.3	+79 48	12.5	R Cancri	8 11.0	+12 2	11.0 *i*
U Cassiop.	40.8	+47 43	8.6*d*	V Cancri	16.0	+17 36	13.0*d*
V Androm.	44.6	+35 6	10.0*d*	RT Hydrae	24.7	− 5 59	8.5
W Cassiop.	49.0	+58 1	9.5	U Cancri	30.0	+19 14	10.0
U Androm.	1 9.8	+40 11	10.0 *i*	X Urs. Maj.	33.7	+50 30	9.8*d*
S Cassiop.	12.3	+72 5	9.5 *i*	S Hydrae	48.4	+ 3 27	8.8*d*
X Cassiop.	49.8	+58 46	9.5	T Hydrae	50.8	− 8 46	9.8*d*
U Persei	53.0	+54 20	8.5*d*	T Cancri	51.0	+20 14	8.5 *i*
W Androm.	2 11.2	+43 50	<13	S Pyxidis	9 0.7	−24 41	12.5*d*
Z Cephei	12.8	+81 13	<13	W Cancri	4.0	+25 39	13.5*d*
S Persei	15.7	+58 8	8.9	X Hydrae	30.7	−14 15	11.5 *i*
RR Cephei	30.4	+80 42	12.0 *i*	Y Draconis	31.1	+78 18	10.5 *i*
R Trianguli	31.0	+33 50	6 5*d*	R Leo. Min.	39.6	+34 58	11.7*d*
W Persei	43.2	+56 34	10.5*d*	RR Hydrae	40.4	−23 34	11.0 *i*
U Arietis	3 5.5	+14 25	11 2*d*	R Leonis	42.2	+11 54	9.3*d*
Y Persei	20.9	+43 50	9.7*d*	Y Hydrae	46.4	−22 33	7.5*d*
R Persei	23 7	+35 20	8.6 *i*	V Leonis	54.5	+21 44	10.0 *i*
T Tauri	4 16.2	+19 18	12.6	R Urs. Maj.	10 37.6	+69 18	12.8 *i*
R Tauri	22.8	+ 9 56	12.0 *i*	W Leonis	48.4	−14 15	10.8 *i*
W Tauri	22.2	+15 49	10.5*d*	S Leonis	11 5.7	+ 6 0	12.2*d*
S Tauri	23.7	+ 9 44	11.0*d*	R Comae	59.1	+19 20	14.0
T Camelop.	30.4	+65 57	8.6*d*	T Virginis	12 9.5	− 5 29	13.7
X Camelop.	32.6	+74 56	11.0 *i*	R Corvi	14.4	−18 42	10.0 *i*
V Tauri	46.2	+17 22	11.7*d*	SS Virginis	20.1	+ 1 19	7.5
R Orionis	53.6	+ 7 59	11.8*d*	T Can. Ven.	25.2	+32 3	10.1 *i*
V Orionis	5 0.8	+ 3 58	<13	Y Virginis	28.7	− 3 52	10.0 *i*
R Aurigae	9.2	+53 28	12.3*d*	T Urs. Maj.	31.8	+60 2	12.4*d*
S Aurigae	20.5	+34 4	9.3*d*	R Virginis	33.4	+ 7 32	11 0
W Aurigae	20.1	+36 49	12.6*d*	RS Urs. Maj.	34.4	+59 2	11.0*d*
T Orionis	30.9	− 5 32	9.6	S Urs. Maj.	39.6	+61 38	9.8*d*
S Camelop.	30.2	+68 45	9.5*d*	RU Virginis	42.2	+ 4 42	9.0
RR Tauri	33.3	+26 19	10.6 *i*	U Virginis	46.0	+ 6 6	9.0 *i*
U Orionis	49.9	+20 10	6.8 *i*	RT Virginis	57.6	+ 5 43	8.5
V Camelop.	49.4	+74 30	<13	RV Virginis	13 2.8	−12 38	<13.5
Z Aurigæ	53.6	+53 18	9.8 *i*	V Virginis	22.6	− 2 39	13.5
X Aurigae	6 4.4	+50 15	8.3 *i*	R Hydrae	24.2	−22 46	8.0 *i*
V Aurigae	16.5	+47 45	9.2	S Virginis	27.8	− 6 41	11.0
V Monoc.	17.7	− 2 9	11.5	T Urs.Min.	32.6	+73 56	13.5*d*
R Monoc.	33.7	+ 8 49	11.6	R Can. Ven.	44.6	+40 2	11.2*d*
S Lyncis	35.9	+58 0	12.0 *i*	RR Virginis	49 6	− 8 43	14.0
X Gemin.	40.7	+30 23	12.6*d*	Z Bootis	14 1.3	+13 59	8.8 *i*
W Monoc.	47.5	− 7 2	10.6	Z Virginis	5.0	−12 50	11.0
Y Monoc.	51.3	+11 22	9.4 *i*	U Urs. Min.	15.1	+67 15	11.5
X Monoc.	52.4	− 8 56	8.3*d*	S Bootis	19.5	+54 16	8.0 *i*
R Lyncis	53.0	+55 28	13.2*d*	RS Virginis	22.3	+ 5 8	14.0*d*
RS Geminorum	55.2	+30 40	10.5 *i*	V Bootis	25.7	+39 18	9.8
V Can. Min.	7 1.5	+ 9 2	<13.5	R Camelop.	25.1	+84 17	9.2*d*
R Gemin.	1.3	+22 52	13.5	R Bootis	32.8	+27 10	11.8 *i*
R Can. Min.	3.2	+10 11	7.5 *i*	V Librae	34.8	−17 14	12.4*d*
RR Monoc.	12.4	+ 1 17	<13.5	U Bootis	49.7	+18 6	11.5
V Gemin.	17.6	+13 17	12.6*d*	RT Librae	15 0.8	−18 21	10.4 *i*
S Can. Min.	27.3	+ 8 32	10.8 *i*	T Librae	5.9	−19 38	12.4*d*
T Can. Min.	28.4	+11 58	12.5 *i*	Y Librae	6.4	− 5 38	12.2
Z Puppis	28.3	−20 57	13.8*d*	S Librae	15.6	−20 2	11.0*d*
U Can. Min.	35.9	+ 8 37	10.0	S Serpentis	17.0	+14 40	10.5 *i*

Approximate Magnitudes of Variable Stars on May 1, 1908—Con.

Name.	R. A. 1900.	Decl. 1900.	Magn.	Name.	R. A 1900	Decl. 1900	Magn.
	h m	° ′			h m	°	
S Coronae	15 17.3	+31 44	9.0	X Draconis	18 6.8	+66 8	10.0 *i*
RS Librae	18.5	−22 33	12.0*d*	RY Ophiuchi	11.6	+ 3 40	11.5 *i*
RU Librae	27.7	−14 59	10.0 *i*	W Lyrae	11.5	+36 38	12.0*d*
X Librae	30.4	−20 50	12.5*d*	SV Herculis	22.3	+24 58	14.0
S Urs. Min.	33.4	+78 58	8.8	T Serpentis	23.9	+ 6 14	12.0
U Librae	36.2	−20 52	10.4*d*	RZ Herculis	32.7	+25 58	13.5
Z Librae	40.7	−20 49	13.4	X Ophiuchi	33.6	+ 8 44	7.0
R Coronae	44.4	+28 28	5.9	RY Lyrae	41.2	+34 34	<14.3
X Coronae	45.2	+36 35	14.0	RW Lyrae	42.1	+43 32	14.0
R Serpentis	46.1	+15 26	9.5 *i*	Z Lyrae	56 0	+34 49	13.3
V Coronae	46.0	+39 52	7.5 *i*	RX Lyrae	50.4	+32 42	<14.0
R Librae	47.9	−15 56	<14	ST Sagittarii	55.9	−12 54	12.8 *i*
RR Librae	50.6	−18 1	12.4*d*	RT Lyrae	57.8	+37 22	13.6*d*
Z Coronae	52.2	+29 32	13.5	R Aquilae	19 1.6	+ 8 5	11.8*d*
RZ Scorpii	58.6	−23 50	11.6	V Lyrae	5.2	+29 30	14.0*d*
Z Scorpii	16 0.1	−21 28	9.8 *i*	RX Sagittarii	8.7	−18 59	9.0
R Herculis	1.7	+18 38	7.8 *i*	RW Sagittarii	8.1	−19 2	9.6
RR Herculis	1.5	+50 46	9.5	S Lyrae	9.1	+25 50	14.0
U Serpentis	2.5	+10 12	8.6 *i*	RS Lyrae	9.3	+33 15	14 2
X Scorpii	2.7	−21 16	12.5	RU Lyrae	9.1	+41 8	<14
W Scorpii	5.9	−19 53	<14	U Draconis	9.9	+67 7	10.0
RX Scorpii	5.9	−24 38	13.6*d*	W Aquilae	10.0	− 7 13	8.4 *i*
RU Herculis	6.0	+25 20	13.0*d*	T Sagittarii	10.5	−17 9	9.0
R Scorpii	11.7	−22 42	13.5	RY Sagittarii	10.0	−33 42	7.0
S Scorpii	11.7	−22 39	14.3*d*	R Sagittarii	10.8	−19 29	9.7
W Coronae	11.8	+38 3	12.6	S Sagittarii	13.6	−19 12	<14
V Ophiuchi	21.2	−12 12	8 5*d*	Z Sagittarii	13.8	−21 7	13.6
U Herculis	21.4	+19 7	11 0 *i*	TZ Cygni	29.8	+28 6	11.0
Y Scorpii	23.8	−19 13	11.8	U Lyrae	16.6	+37 42	12.0
SS Herculis	28.0	+ 7 3	11.9*d*	T Sagittae	17.2	+17 28	9.0
S Ophiuchi	28.5	−16 57	13.4*d*	TY Cygni	29.8	+28 6	13.5
T Ophiuchi	28.0	−15 55	10.4*d*	R Cygni	34.1	+49 58	11.0 *i*
W Herculis	31.7	+37 32	13.6	RV Aquilae	35.9	+ 9 42	13.5
R Draconis	32.4	+66 58	7.4	RT Aquilae	33.3	+11 30	8.6
RR Ophiuchi	43.2	−19 17	11.8*d*	RT Cygni	40.8	+48 32	12.0
S Herculis	47.4	+15 7	11.8*d*	TU Cygni	43.3	+48 49	9.8
RV Herculis	56.8	+31 22	11.0	X Aquilae	46.4	+ 4 13	<13.5
R Ophiuchi	17 2.0	−15 58	12.4*d*	χ Cygni	46.7	+32 40	5.0 *i*
RT Herculis	6.8	+27 11	12.0	RR Aquilae	52.4	− 2 11	9.3 *i*
Z Ophiuchi	14.5	+13 7	13.4	RS Aquilae	53.7	− 8 9	12.8
RS Herculis	17.5	+23 1	11.6*d*	Z Cygni	58.6	+49 46	12.4*d*
RU Ophiuchi	28.1	+ 9 30	13.2*d*	X Cephei	21 3.6	+82 40	12.0 *i*
RS Ophiuchi	44.8	− 6 40	11.3	T Cephei	8.2	+68 5	7.0 *i*
RT Ophiuchi	51.8	+11 11	<13	S Cephei	36.5	+78 10	7.0
T Draconis	54.8	+58 14	9.3 *i*	S Lacertae	22 24.6	+39 48	11.0 *i*
RY Herculis	55.4	+19 29	10.3*d*	R Lacertae	38.8	+41 51	9.0 *i*
V Draconis	56.3	+54 53	10.6*d*	V Cassiop.	23 7.4	+59 8	7.5
T Herculis	18 5.3	+31 0	9.2 *i*	Z Cassiop.	39.7	+56 2	11.0 *i*
W Draconis	5.4	+65 56	9.8 *i*	R Cassiop.	53.3	+50 50	12.5*d*
				Y Cassiop.	58.2	+55 7	9.5 *i*

The letter *i* denotes that the light is increasing, the letter *d* that the light is decreasing, the sign <, that the variable is fainter than the appended magnitude.

The magnitudes given above have been compiled by Mr. Leon Campbell of the Harvard College Observatory, from observations made at the Vassar, Mt. Holyoke, and Harvard Observatories.

The Variable Star 31.1907 was found to be bright, magnitude 10.5 on April 29 at 13^h G. M. T. In the course of the next two hours, the increase any, was only slight. Observations were then stopped by clouds. At the last maximum it remained above the magnitude 12.5 for about five days.

EDWARD C. PICKERING.

Astronomical Bulletin, No. 327.
Harvard College Observatory,
Cambridge, Mass.

New Variable 7.1908 Monocerotis.—In *A. N.* 4245 Professor W Ceraski announces a new variable in Monoceros. Its position is

1855.0	$\alpha =$	7^h	22^m	07^s	$\delta = -4°\ 0'$
1900.0		7	24	21	$-4\ 5$

On the photographs its brightness is estimated as follows:

1899 Feb.	6	<12½ mag.	(invisible)
1900 Mar.	19	<12½ "	"
1901 "	15	<12 "	"
1903 "	16	10½ "	
1905 "	2	9½ "	
1907 Feb.	16	10½ "	
1908 Mar.	4	11½ "	

The Light Curve of RV Tauri.—In *A. N.* 4243 Mr. S. Enebo gives an interesting curve, drawn from about 100 of his own observations of this variable, in the interval from August 27, 1906 to March 1908. The curve as shown in Figure 1 appears to be that of a variable of the β Lyræ type, with a period of 78.57 days, superposed upon another of much longer period. The

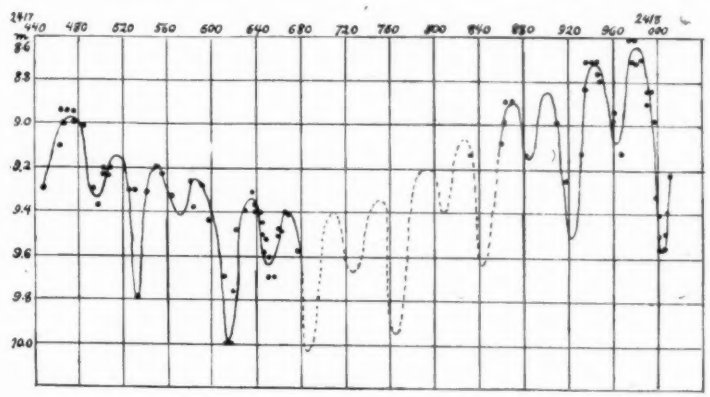


FIGURE 1

magnitude of the star at its maximum diminished from $8^m.9$ on September 18, 1906 to $9^m.4$ on April 2, 1907. During the winter of 1907-08 it increased to about $8^m.6$. The two minima of β Lyræ type are nearly symmetrical, one

having more than double the range of the other. Mr. Enebo gives for the elements of the β Lyræ-variation

Chief minimum	= 2417609.6 Gr. M. T.		$+ 78^d.57$ E
Secondary "	= 2417648.9	"	+ 78 .57 E
Maximum	= 2417629.2	"	+ 39 .285 E.

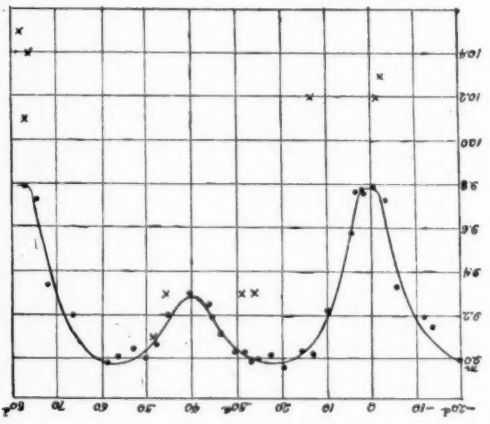


FIGURE 2.

Figure 2 shows the curve when all the observations have been reduced so as to make the normal brightness at maximum 9.0.

GENERAL NOTES.

Jupiter's Eighth Satellite. Continued observations of the moving object observed near Jupiter by Mr. Mellotte, of the Greenwich Observatory staff, has confirmed the first supposition that it was a new satellite. The body has also been photographically recorded at the Lick Observatory and by Dr. Wolf at Heidelberg. It is slightly brighter than J. VII. Details are given of the measured positions relative to Jupiter from 1908 January 27, to March 23, and on analysis it is found that these positions may be satisfied on the assumption that the new satellite has a retrograde orbital motion; the co-ordinates of the pole of its orbit plane are R.A. = 334° 48′; N. decl. = 56° 44′; distance from Jupiter = 0.24 of an astronomical unit; daily motion about Jupiter 0°.266. It passed the minor axis of the apparent ellipse on February 18.

KNOWLEDGE AND SCIENTIFIC NEWS.
May, 1908.

Spectrum and Brightness of Nova Persei. Professor J. Hartmann describes the conclusions he has been able to formulate from his recent observations of the body now representing Nova Persei. For this work he designed a new spectroscope having a collimator objective of 40 mm. aperture and 60 c. m. focal length, made of ultraviolet glass, two right and left handed quartz half-prisms of 30°, and a camera objective of quartz, 40 mm. aperture and 32 c. m. focal length. With this apparatus attached to the 80 c. m.

refractor he obtained an excellent photograph of the Nova spectrum on October 15 and 18, 1907, with a total exposure of 8¼ hours, the magnitude of a star being 11.4. The spectrum obtained was found to be very similar to that of the Wolf Rayet star B. D. 35° 4001. The line λ 4688 is the strongest in both spectra, while the fairly strong Wolf Rayet line λ 4618 is comparatively faint in the Nova. The nebula lines, which were very strong during the first part of the period of decline, are now very faint or absent.

ASTRONOMISCHE NACHRICHTEN.

February 8, 1908.

The Calendar. In examining the question as to the frequency with which any given day of any month will fall on Sunday, or other day of the week, a very curious general law was discovered—a law not hitherto noticed, so far as I know.

To illustrate:—the fourth of March fell on Sunday in 1821, then again in 1827, 1832, 1838, and 1849; the intervals being in succession 6, 5, 6, and 11 years. From 1849 to 1877 the like intervals are found. And these intervals would follow in the same order for all time, except for the fact that centesimal years which are not divisible by 400 are not leap years, as 1700, 1800, 1900, 2100, 2200, &c. Thus three times in 400 years the regular intervals are interrupted for a period of twelve or fifteen years, but the old order is soon restored.

THE RULE IS UNIVERSAL—*Every day of the year falls on every day of the week four times in 28 years.* The intervals for the successive years would always be 6, 5, 6, and 11, but for the fact above stated that the 29th of February is dropped three times in 400 years. Astronomers will see that these 28 years stand for the *Solar Cycle*.

It may be well to pursue this subject a little further. Following 1877, the 4th of March fell on Sunday in 1883, 1888, 1894, 1900 and 1906, the intervals being 6, 5, 6, 6, and six years. If 1900 had been a leap year, the regular intervals would have appeared.

After the adoption of the constitution of the United States, (the election for President being in leap years,) the date for the inauguration would be the 4th of March next following a leap year. The first inauguration day which fell on Sunday was in 1821, at Monroe's second term.

In 1849 it occurred again, at Taylor's inauguration, and again in 1877, when Hayes became President. It will not occur again until 1917, nine years in the future. After that year it will occur six times at intervals of 28 years, the last of the six being in the year 2085. Following that year, it will not again take place for forty years, the same interval as that between 1877 and 1917. Whenever this 28-year period includes a centesimal year which is not a leap year, as 1900, 2100, &c., the period is increased to forty years, neither more nor less.

In 1685 March 4th fell on a Sunday, and counting forward for a few years into the next century, the successive intervals are 6, 5, 7, 5, 6, 11. Four centuries later, the like intervals will occur, but not before. This series of years includes the year 1700. The 4th of March 1781 was Sunday, and counting forward a few years into the following century, we find the series of intervals to be 6, 5, 6, 6, 6, 11. Here we pass by the year 1800. Four centuries later the same intervals will occur again. In like manner if we pass over the year 1900, we find the same series which we had in passing over

1800. But in passing over 1600, 2000, &c , which are leap years, only the regular four numbers are found, viz. 6, 5, 6, 11. All possible changes and variations take place in 400 years. The first of January 1600, 2000, &c., will fall on the same day of the week, and so on forever. So any day of any month in the year 1601 will fall on the same day of the week, as the like day of 2001. And the same law holds good for each successive year of the four hundred,—a degree of regularity which is somewhat remarkable and which would hardly have been considered probable before the investigation was made.

R. W. McFarland.
Emeritus Professor of Civil Engineering, Ohio State University.

Physical Phenomena of the Transit of Mercury. I wish to acknowledge the kind attention of astronomers, observing the last transit of Mercury, to the phenomenon of the complete illumination of the planet's disk when half of it was projected on the chromosphere. The observations as a whole, while not as decisive as could be wished, seem to point to the reality of the phenomenon as originally observed by me May 6, 1878. The illumination mentioned seems to be a very delicate phenomenon, and perhaps visible only at a critical moment. I should therefore deem it worth the while of observers to make the attempt to see this phenomenon at future transits of the planet Mercury.

Our observations at the Philadelphia Observatory failed to disclose the phenomenon mentioned. No suitable time-pieces being available, the entire attention was given to the physical phenomena. Dr. Paul R. Heyl and myself observed through the 8″ telescope, and, alternating in our view of the planet, could not see any suggestion of the full illumination of the planet when half way off the limb of the Sun. The seeing was not good, the Sun's limb showing considerable atmospheric disturbance. I have since realized that I made a mistake in endeavoring to have two observers use the same telescope for verification of the phenomenon mentioned. It is quite certain that only one observer can use the telescope to advantage in such a trial, although I had felt rather sure that, if the seeing were good enough, two observers could verify the phenomenon through the same instrument.

While the planet was approaching the solar edge I noticed a grayish illumination of the following semi-disk of Mercury, and, without describing the phenomenon, asked Dr. Heyl to see if he noticed any difference of illumination in the planet. He reported no difference, and at subsequent trials could not see what I saw then, and subsequently. My faith in the reality of the semi-illumination of the planet while approaching the solar edge was somewhat staggered by this negative decision of my friend, and hence I made up my mind that I would make no report of the phenomenon, although I saw this grayish illumination change position as the planet approached the limb of the Sun.

On reading the detailed description of the observations of Professor G. A. Hill, as reported by Dr. W. S. Eichelberger in No. 601 of the *Astronomical Journal*, I decided to state that I had verified, in its essential elements, Professor Hill's physical observations of the dull illumination of the following half of the planet early in the approach to the limb, and the change of this illumination to the preceding side. It was all so evident and yet so puzzling to me, and so completely unverified by the colleague who so kindly agreed to try to verify anything seen, that but for the excellent report of Dr. Eichelberger the appearance would never have been mentioned.

It seems worth while to say to observers of the transit of Mercury that it is utterly useless to endeavor to see a *ring* about the planet. It seems quite certain that none such exists, and that it is only possible to see the gray illumination of the disk just before its approach to the solar limb, and at its midway position on the limb.

In conclusion I would draw attention to the negative results of so excellent a physicist and observer as Dr. Heyl, as against the positive observation of the very curious phenomenon by Professor G. A. Hill and myself. Should there not be some plan of applying delicate standard tests to the eyes of astronomical observers so as to decide scientifically and definitively who ought to see and who ought not to see the different classes of celestial phenomena? MONROE B. SNYDER.

Philadelphia Observatory.
May 14, 1908.

Volume II of the Astrophysical Observatory of the Smithsonian Institution. We are in receipt of the second volume of the Annals of the Astrophysical Observatory of the Smithsonian Institution at Washington, D. C., under the direction of C. G. Abbot, with F. E. Towle Jr. as Aid.

This quarto volume of 245 pages, 44 tables and 29 fine illustrations is a masterful piece of work in its line. Our frontispiece is a reproduction of a photograph of the Observatory building.

This Astrophysical Observatory was founded through the efforts of the Institution's late secretary, Professor S. P. Langley, who was its director until his death. The work described in this volume is a continuation of researches on the relations of the Sun to climate and life on the Earth in which the brilliant Langley was a pioneer investigator. The writer well remembers the long and earnest work of this great man in the hope of laying a certain and secure foundation for successful study of the radiation of the Sun that there might be reliable means of forecasting climatic conditions for some time in advance. That his methods of work were scholarly, certain and exact, the best talent of today does not seriously question.

This new volume shows plainly that the hope of Langley may yet be largely realized in the years to come, for it contains apparently careful and comparable measurements of the solar radiation extending over a considerable period of past time, and these seem to indicate that the Sun's radiation alters in its intensity from time to time and that these alterations are sufficient to effect the temperature of the Earth very appreciably. Such approaches to these delicate physical questions are very encouraging for practical and hopeful weather predictions when the fundamental facts are better known.

The Red Spot on Jupiter. An interesting report on the condition and motion of the Red Spot on the planet Jupiter appears in the last number of the *Journal* of the British Astronomical Association. The great south tropical disturbance in connection with the Red Spot has claimed the attention of the Jupiter section of the society for some time past. The points of observation have been to note the mode of transference of the dark matter from the following to the preceding side of the Red Spot hollow; to notice the interval of time elapsing between the arrival of the preceding end of the disturbance and the following end of the Red Spot, and the appearance of the dark matter on the tropical zone above the preceding 'shoulder', to notice also the effect of the conjunction on the motion of the Red Spot.

The results of recent observations seem to show that portions of the disturbed area are accelerated as they approach the Red Spot hollow; that at previous conjunctions the dark matter seemed to be passing around the south side of the Red Spot; that the length of the disturbance has varied considerably since the beginning of the apparition, and that the rotation period of the Red Spot is undoubtedly less than during the earlier part of the apparition.

The variety of interesting details in the report of this work on the surface markings of Jupiter in the vicinity of the Red Spot is very useful matter in the full record that this section of the society is making. We earnestly hope that like painstaking observation and continued study of this planet may be done as well on this side of the ocean. In this work, as in all other, organized effort pays most largely.

Recently Discovered Manuscript of Archimedes. In the May number of the *Bulletin* of the *American Mathematical Society*, page 382, occurs an account of a recent discovery of a new manuscript of Archimedes, by Charles S. Slichter. The information that he gives of the discovery is derived from two important accounts published by Professor J. L. Heiberg. "One account is printed in Volume 42 of *Hermes* which contains the Greek text of the last treatise of Archimedes which is recovered nearly complete in the newly found manuscript. A German translation of the Greek text, and an interesting commentary by Zeuthen, is printed by Heiberg in the Bibliotheca Mathematica Vol 7. page 321."

This article is valuable for the compact statement of the many points of interest that the newly found manuscript reveals. Mathematicians will be eager to get full knowledge of it from these very helpful sources.

As an illustration of the method of treatment of this new matter we give the final paragraph:—

Archimedes makes it clear that he is using this method (described before) to discover the theorems, and that it is not founded on demonstration. But in speaking of spheres as "filled up" by circles, and of surfaces as "made up" of lines he is not misusing the method of infinitesimals, nor treading on dangerous ground. In fact the elements which "fill up" the magnitudes are always so taken by Archimedes, that the process can be immediately satisfied by an exhaustive proof. From this point of view his scheme may be regarded as analogous to the modern method of infinitesimals when founded on the doctrine of limits. There is a normal and systematic procedure, although tedious and laborious, for converting the mechanical proofs into exhaustive proofs. Therefore Archimedes may be regarded as having taken the decisive step in founding a method which in essential respects is that of the Integral Calculus. If he had been like many modern mathematicians, he would have omitted the exhaustive proofs altogether, but would have added to each of his mathematical demonstrations a set phrase like this: "It is easy to see that an exhaustive proof may be constructed in the usual manner. This is left to the reader."

Correlation of Stellar Characters. In volume 66 p. 445 of the *Monthly Notices* appeared a paper of much interest which presented in full, as far as carried, considerations on the correlations of stellar characters by Winifred Gibson and Karl Pearson of London.

In volume 58, No. 5, p. 415 this interesting correlation of the stellar data

is continued, covering a space of 33 pages with six illustrative diagrams. We have room only for the conclusions drawn from this extended and painstaking work.

Conclusions.—While we are fully aware how badly the mere statistician may stumble in dealing with astronomical data, we still think that the general relationships shown by the statistical correlation constants may be of value to astronomers. They serve to indicate the directions in which closer relationships may be found, and where, possibly, more effective classifications may be made. The values given in the accompanying general scheme are certainly not final, but we do not think that they give at all misleading values, or values really far from the truth. There remains much to be done, and the scheme indicates some of the values which yet remain to be found. While parallax touches one element only of position, direction, as indicated by the usual stellar coördinates is a second; this "position" in the narrower sense has not been discussed in the present paper. We have purposely included it in the table, to indicate that we have not overlooked those position correlation problems to which the astronomer is now turning, and to which approach is possible from more than one direction. We hope later to deal with the question of correlation of stellar characters and position. Taken as a whole, we are, we think, compelled to conclude that the associations between parallax, proper motion, and magnitude are considerably less than we should anticipate if we hypothecated any approach to a uniform distribution of stars with a system of random velocities throughout space. The existence of correlations between color and spectral class, not only with magnitude, but with parallax and proper motion, suggests, if it does not demonstrate, that chemical constitution and luminosity are dependent in some manner not only on spatial distribution, but on velocity in space.

Correlation of Stellar Characters.

	Color.	Spectral Class.	Magnitude.	Parallax.	Proper Motion.	Position
Color	1.00	.71	.30	?	?	?
Spectral class	.71	1.00	.69(.43, .54*)	.36	.36	?
Magnitude	.30	.69(.43, .54*)	1.00	.30	.35	?
Parallax	?	.36	.30	1.00	.30	?
Proper motion	?	.36	.35	.39	1.00	?
Position	?	?	?	?	?	1.00

* According to Pickering's and Lockyer's classifications respectively.

Astronomy at Marsovan, Turkey. A recent letter from Dr. A. G. Sivaslian, of Marsovan, Turkey says that he has quite completed the manuscript of a text-book on Astronomy which he has been preparing for some months past, references to which have already been made in this publication.

In preparing the star charts he says: "I tried to prepare star charts for every third month of the year, similar to those published in the POPULAR ASTRONOMY. Now I have changed my mind and am preparing a single chart which will contain stars down to 30° south declination. In its preparation I have copied the northern constellation from Poole Brothers' charts published in POPULAR ASTRONOMY, and condensed the southern constellations to the same scale in declination as the northern constellations. It will be a circular chart of about eight inches in diameter and will be folded in the middle.

The transit of Mercury was the most interesting astronomical event since I came here. According to calculation it was to begin at 12:44 p. m. In the morning after chapel exercises I gave a short talk to the students about the transits of Mercury and Venus, and told them about the transit of that day, but there was not much prospect of our seeing it. It was cloudy all the morning. At noon I went to dinner; at 12:30 there were some breaks in the clouds so I ran to the college. Our three-inch telescope was already in its place inside of a south window. At 12:40 there was a break very near to the Sun, so I pointed the telescope to the Sun and waited. At 12:45 I saw the Sun the first time and the planet was already on the Sun just past the second contact. I could see the Sun only about two minutes. I took the telescope down to the campus with the hope that there might be other breaks, but very soon thicker clouds gathered and it began to rain. So I had to take the telescope into the building again. After 3:00 p. m. the sky cleared a little, and I was able to show the planet to all the students class by class. Toward four o'clock I took the telescope to the girls' school and there Miss Willard showed it to her astronomy class."

Brilliant Meteors seen from Dumont, Iowa. Saturday evening April 18th, 1908 at 10:30 I saw a meteor fall from high in the northeastern heavens towards the northwest continuing to fall until quite near the horizon where it became extinct.

There was a full Moon and as the meteor fell it looked nearly as large as the full Moon. As it fell it left a long glowing trail and numerous sparks behind it; it lit everything up with a beautiful white light making it nearly as light as day. I think that the meteor was falling about twenty-five or thirty seconds I did not hear any unusual sound.

On August 14, 1907 at eleven o'clock I witnessed a meteor fall from high in the southeastern heavens toward the northwest. It was a very dark night there being no Moon and as that meteor fell it looked larger than a full Moon. I did not hear any unusual sound. It fell slower than any other meteor I have seen. I think it must have been falling more than a minute. It fell from very high in the southeast to very near the horizon in northwest. A beautiful white light was shed around making it very nearly as bright as sunlight. It left a long glowing trail after it, but no sparks.

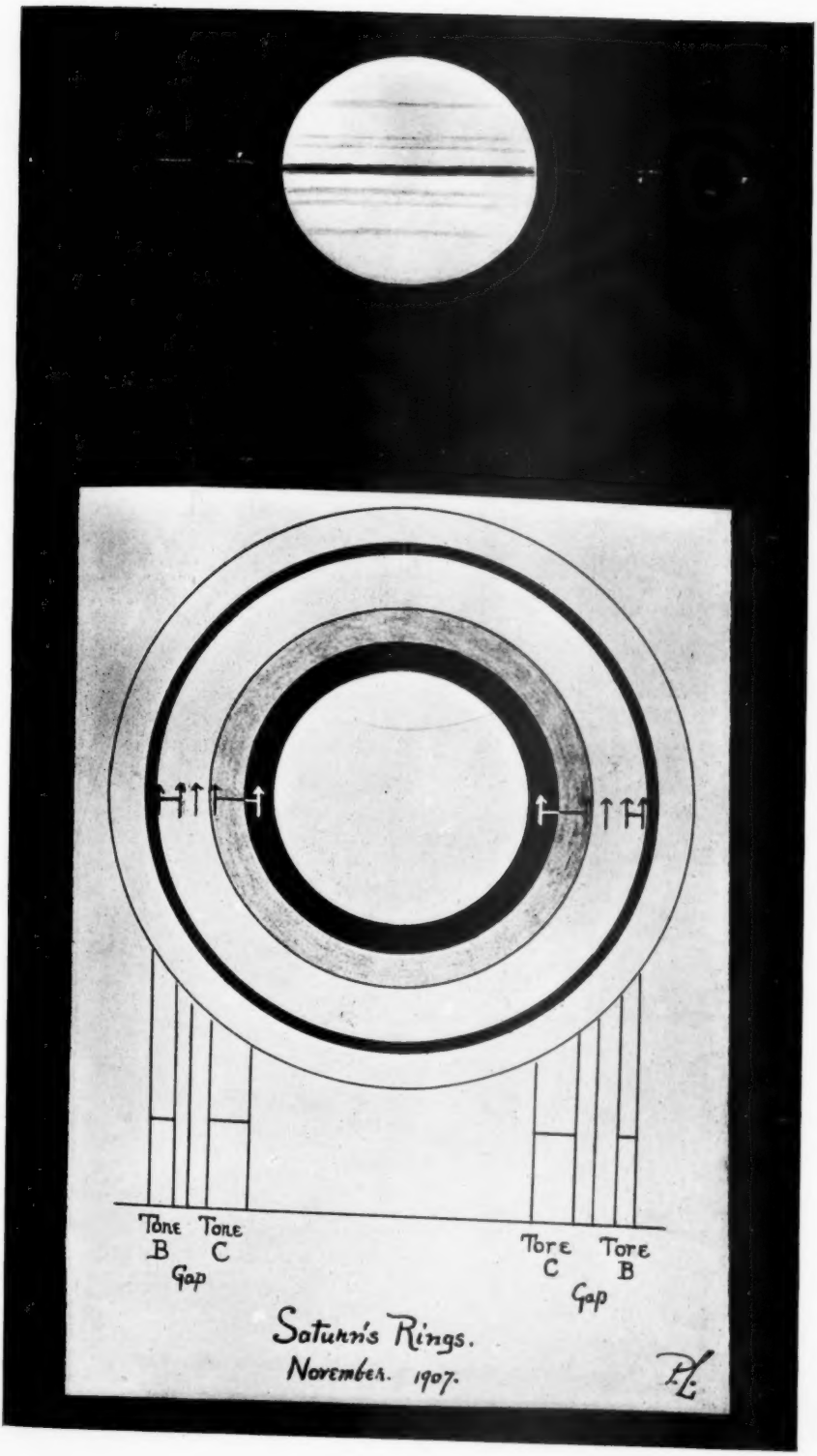
ETHEL F. LOOMIS.

Dumont, Iowa.

Mr. Lowell's Illustration of Saturn's Rings. The accompanying plate in this number is to replace that of the March one in order to bring out the features of the Saturnian ring-system observed at Flagstaff in 1907, especially of its shadow on the ball and of that shadow's black core. Owing to the small size of the first cut and of the general difficulties of reproduction it failed to show these peculiarities, the detection of which constituted an essential part of the paper—and so we run this time an unusually large cut at Mr. Lowell's request even at the expense of the looks of the page.

New Earthquake Theory. The new theory of earthquakes and volcanoes is objectionable. I will give a few reasons for my opinion.

At a depth of two miles the temperature is 212° more or less according to ocality. A geyser is simply a deep hole or well reaching down to the hot rocks


Tone B
Tone C
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Saturn's Rings.
November. 1907.
P.L.

with a stream of water entering its sides. While the water in the bottom is heating the well is filling, the pressure becoming greater on the heated water in the bottom, circulation being cut off so the heat can not escape, the much heated water in the bottom is converted into steam forcing the water above out in a splendid display, these actions taking place at regular intervals so long as the heat is applied at the bottom and the water comes in.

In volcanoes the crust of the Earth has a vent or opening down to the liquid interior which remains some distance below sea level the world over except raised by water entering this opening far beneath the surface of the molten mass through another fissure or vent connected with the surface water of the ocean or more likely a lake or other intake of a high altitude far above the surface of the molten mass which would give sufficient pressure to force the water into the volcanic tube raising the molten mass above the water gradually until an eruption would take place which would throw out a part or all of the contents. The molten matter rising again from beneath the same operation would be repeated in duration of time according to the water pressure and supply.

We have a fine illustration of volcanoes in some California wells, say one 300 feet deep, the water rises 200 feet in it or within 100 feet of the surface, a small pipe is put down to the bottom of the well through which air is forced causing the water and air to flow out at the top by turns, the weight of the water preventing the air coming through it in small bubbles but holds it in larger volumes which fill the caisin of the well for a number of feet, thus showing how the molten mass in a volcano might float on a large volume of steam. This would account for the different heights at which lava stands in different volcanoes.

The adherents of the new theory seem to have overlooked the fact of the land and mountains being full of water within a few feet of the surface at least as attested by the many springs and seeps forming into rivers carrying the water overland to the sea, thus giving a water pressure of from a few feet to several miles according to elevation under the land greater than it has under the sea. Such a theory would have us believe a light weight would raise a heavier one. Again the idea of a light liquid like water or steam penetrating into a heavier molten liquid such as the interior of the Earth must be would be like a cork sinking in water. If water never had existed on the Earth there would be mountains and earthquakes just the same, but no violent ejections of volcanoes and perhaps none at all.

The lay of the land and mountains certainly does not or never has depended on the lay of the seas as is claimed in paragraph one and two, but the oceans depend on the lay of the land for their locality.

As the Earth's crust becomes thicker its surface becomes more uneven by the shrinkage of the interior, some coasts going down and some lifting up as nearly all parts have been submerged and elevated many times by turns.

If the mountains were formed by expansion there would be great gaping fissures on their divides constantly widening and not as we find, the strata shoved together, broken up and twisted and mixed in more or less confusion. As to gravity being less on the elevated portions of the Earth as stated in paragraph 12 ; it does not necessarily depend on a light or porous substance beneath for as we leave the main body of the Earth gravity decreases.

G. M. Crowl.

Covina, California.

Young and Jackson's Elementary Algebra. There are things in this new book that will commend themselves to experienced teachers in Algebra. They pertain to the present general movement for better teaching in Algebra. This fact appears in the following ways:

1. The book gives little of mathematical theory, but much of the utility of the subject.
2. It aims at logical and practical values in mental drill.
3. It makes the properties of the equation a central thought in the plan of the book.
4. The graph is freely used more as a means of illustration than as an analytic method.

The usual courses in Algebra including quadratics are given in this book, and the evidences of a teacher's skill are everywhere present in the detail of it.

Revue Canadienne, a monthly publication of 44 years standing, has this year passed under the control of professors of Laval University, Montreal. It appears to be a useful exponent of University life in general.

Longitude of Observatory at Mare Island Navy Yard. The longitude of the Observatory at Mare Island Navy Yard has been recently determined by electrical time signals from Lick Observatory at Mount Hamilton, California, R. H. Tucker and R. F. Sanford being in charge at the last named Observatory. The Lick Observatory longitude was determined in 1888 and was found to be $8^h\ 6^m\ 34^s.81$ west of Greenwich. The difference between Mt. Hamilton and Mare Island is $2^m\ 30^s.74$.

The longitude of the Observatory at Mare Island is $8^h\ 9^m\ 5^s.55$ west of Greenwich. We notice an interesting feature in this determination. It was that at one station mean-time signals were used in the exchange while at the other sidereal-time signals were used. The large relative rate of the latter signal is spoken of as an advantage in the determination.

Haustein's Skeleton Models and Goniostat are helpful as classroom devices for practical demonstrations in drawing arithmetic, perspective shadows, stereometry, axiometry, crystallography and astronomy.

Astronomy by Postcard. Dr. E. D. Roe, Jr., of Syracuse, N. Y., has sent us a number of postcards having photographs of instruments, sunspots and his Observatory. The illustrations were fairly good for photographs taken in this way.